Towards Foundations for Traditional Oriental Medicine

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Introduction. Traditional oriental medicine incorporates hundreds (maybe even thousands) of years of experience. Some parts of it have already been described in precise terms and used in the West (see, e.g., [1]). However, there are still methods and ideas in Oriental medicine, such as acupuncture, moxibustion, massage, acupression, etc., that seem to work well for various diseases but that are not yet formalized and not yet widely used. It is, therefore, desirable to formalize these methods.

So far, the main efforts were in designing computer-based expert system that would incorporate the rules and techniques used by experts (see, e.g., [5] and references therein). In this paper, we show that uncertainty formalisms can be used not only to *describe* these rules, but also to *justify* them, i.e., to provide the *foundations* for traditional oriental medicine.

We need a family of sets. In all above-enumerated techniques, to cure a disease or to improve a patient's condition, we apply a certain action (a needle, a massage, etc.) to one of the special points on the patient's body.

The main problem is to find the most appropriate points, activating which will lead to the best possible effect on a patient. Different points may be optimal for different diseases, so for each patient, we have, in general, a *set* of points which are optimal with respect to different diseases. Therefore, in mathematical terms, for each patient, our goal is to find the *set* of points activation in which leads to the best cure.

Since people are different, this optimal set of points can vary from a patient to a patient. With this difference in mind, our goal is, therefore, to find a *family* of sets which would correspond to different patients. Here, by a *family*, we mean a *finite-dimensional* family, i.e., a family in which each element can be described by specifying values of finitely many parameters.

We want to find the first two approximations to the optimal family of sets. Of course, without having a clear physical understanding of how different methods like acupuncture work, we cannot get the *exact* family of optimal sets. Our goal is to use the first principles (namely, the natural geometric symmetries) to get a good *approximation* to the desired family of sets.

Our first goal is to get a *first* (crude) approximation. After we get the *first* approximation, our next goal will be to get a *better* approximation. For example, in the first approximation, which (roughly speaking) corresponds to computing *approximately* best cures, we may get too many points which are, in this approximation, reasonably good. In the next approximation, we may want to improve this picture by selecting a *subset* of each first-approximation set of points, a subset which consists of those points which are not only *approximately* best, but *truly* best. These subsets will give us the *second* approximation to the optimal family of sets.

What is "optimal"? Our goal is to find the *best* (*optimal*) family of sets. When we say "optimal", we mean optimal w.r.t. to some *optimality criterion*. When we say that some *optimality criterion* is given, we mean that, given two different families of approximating sets, we can decide whether the first one is better, or that the second one is better, or that these families are of the same quality w.r.t. the given criterion. In mathematical terms, this means that we have a *pre-ordering relation* $\preceq$ on the set of all possible finite-dimensional families of sets.

One way to approach the problem of choosing the "best" family of sets is to select *one* optimality criterion, and to find a family of sets that is the best with respect to this criterion. The main drawback of this approach is that there can be different optimality criteria, and they can lead to different optimal solutions. It is, therefore, desirable not only to describe a family of sets that is optimal relative to *some* criterion, but to describe *all* families of sets that can be optimal relative to different natural criteria. In this paper, we are planning to implement exactly this more ambitious task.

Examples of optimality criteria. Pre-ordering is the general formulation of optimization problems in general, not only of the problem of choosing a family of sets. In general optimization theory, in which we are comparing arbitrary *alternatives* A, B, ..., from a given set $\mathcal{A}$, the most frequent case of such a pre-ordering is when a *numerical criterion* is used, i.e., when a function $J : \mathcal{A} \to R$ is given for which $A \preceq B$ if and only if $J(A) \le J(B)$.

Several natural numerical criteria can be proposed for choosing the best family of sets: if we approximate the actual set of possible values X by an element $\tilde{X}$ from the chosen family, then we can measure the quality of the approximation by computing the Lebesgue measure of the difference between the two sets, or by computing the Hausdorff distance between these two sets. As an optimality criterion, we can, e.g., choose the *average* value of this quality measure (average in the sense of some natural probability measure on the class of all problems).

Alternatively, we can fix a class of problems, and take the *largest* (worst-case) value of the quality measure for problems of this class as the desired (numerical) optimality criterion.

For "worst-case" optimality criteria, it often happens that there are several different alternatives that perform equally well in the worst case, but whose performance differ drastically in the average cases. In this case, it makes sense, among all the alternatives with the optimal *worst-case* behavior, to choose the one for which the *average* behavior is the best possible. This very natural idea leads to the optimality criterion that is *not* described by a numerical optimality criterion $J(A)$: in this case, we need *two* functions: $J_1(A)$ describes the worst-case behavior, $J_2(A)$ describes the average-case behavior, and $A \preceq B$ if and only if either $J_1(A) < J_2(B)$, or $J_1(A) = J_1(B)$ and $J_2(A) \le J_2(B)$.

We could further specify the described optimality criterion and end up with *a* natural criterion. However, as we have already mentioned, the goal of this paper is not to find *a* family of sets that is optimal relative to some criterion, but to describe *all* families of sets that are optimal relative to some natural optimality criteria. In view of this goal, in the following text, we will not specify the criterion, but, vice versa, we will describe a very general class of *natural* optimality criteria.

So, let us formulate what "natural" means.

The criterion must be invariant. Problems related to geometric sets often have natural *symmetries*. Locally, a body surface is a plane (R^2). So, sets that we are talking about are sets in R^2. For such sets, there are two natural symmetries:

First, if we *change the starting point* of the coordinate system from the previous origin point $O = (0,0)$ to the new origin O' whose coordinates were initially $a = (a_1, a_2)$, then each point x with old coordinates (x_1, x_2) gets new coordinates $x_i' = x_i - a_i$. As a result, in the new coordinates, each set $X \in A$ from a family of sets A will be described by a "shifted" set $T_a(X) = \{x - a \mid x \in X\}$, and the family turns into $T_a(A) = \{T_a(X) \mid X \in A\}$. It is reasonable to require that the relative quality of the two families of sets do not depend on the choice of the origin. In other words, we require that if A is better than B, then the "shifted" A (i.e., $T_a(A)$) should be better than the "shifted" B (i.e., that $T_a(B)$).

Second, the choice of a *rotated* coordinate system is equivalent to rotating all the points ($x \to R(x)$), i.e., going from a set X to a set $R(X) = \{R(x) \mid x \in X\}$, and from a family A to a new family $R(A) = \{R(X) \mid X \in A\}$. It is natural to require that the optimality criterion is invariant w.r.t. rotations, i.e., if A is better than B, then $R(A)$ is better than $R(B)$.

The criterion must be final. If the criterion does not select any family as an optimal one, i.e., if, according to this criterion, none of the families is better than the others, then this criterion is of no use in selection.

If the criterion considers several different families equally good, then we can always use some other criterion to help select between these "equally good" ones, thus designing a two-step criterion. If this new criterion still does not select a unique family, we can continue this process until we arrive at a combination multi-step criterion for which there is only one optimal family.

Therefore, we can always assume that our criterion is *final* in the sense that it selects one and only one optimal family.

Let us describe these conditions of invariance and finality in precise mathematical terms. (With other potential applications in mind, we will try, whenever possible, to make the definitions as general as possible.)

Definition 1. *Let $g : M \to M$ be a 1-1-transformation of a set M, and let A be a family of subsets of M. For each set $X \in A$, we define the result $g(X)$ of applying this transformation g to the set X as $\{g(x) \mid x \in X\}$, and we define the result $g(A)$ of applying the transformation g to the family A as the family $\{g(X) \mid X \in A\}$.*

Definition 2. *Let M be a smooth manifold. A group G of transformations $M \to M$ is called a Lie transformation group, if G is endowed with a structure of a smooth manifold for which the mapping $g, a \to g(a)$ from $G \times M$ to M is smooth.*

We want to define $r-$parametric families sets in such a way that symmetries from G would be computable based on parameters. Formally:

Definition 3. *Let M and N be smooth manifolds.*

- *By a multi-valued function $F : M \to N$ we mean a function that maps each $m \in M$ into a discrete set $F(m) \subseteq N$.*
- *We say that a multi-valued function is smooth if for every point $m_0 \in M$ and for every value $f_0 \in F(m)$, there exists an open neighborhood U of m_0 and a smooth function $f : U \to N$ for which $f(m_0) = f_0$ and for every $m \in U$, $f(m) \subseteq F(m)$.*

Definition 4. *Let G be a Lie transformation group on a smooth manifold M.*

- *We say that a class A of closed subsets of M is $G-$invariant if for every set $X \in A$, and for every transformation $g \in G$, the set $g(X)$ also belongs to the class.*
- *If A is a $G-$invariant class, then we say that A is a finitely parametric family of sets if there exist:*
  - *a (finite-dimensional) smooth manifold V;*
  - *a mapping s that maps each element $v \in V$ into a set $s(v) \subseteq M$; and*
  - *a smooth multi-valued function $\Pi : G \times V \to V$*

  such that:
  - *the class of all sets $s(v)$ that corresponds to different $v \in V$ coincides with A, and*
  - *for every $v \in V$, for every transformation $g \in G$, and for every $\pi \in \Pi(g, v)$, the set $s(\pi)$ (that corresponds to π) is equal to the result $g(s(v))$ of applying the transformation g to the set $s(v)$ (that corresponds to v).*
- *Let $r > 0$ be an integer. We say that a class of sets B is a $r-$parametric class of sets if there exists a finite-dimensional family of sets A defined by a triple (V, s, Π) for which B consists of all the sets $s(v)$ with v from some $r-$dimensional submanifold $W \subseteq V$.*

Definition 5. *Let $\mathcal{A}$ be a set, and let G be a group of transformations defined on $\mathcal{A}$.*

- *By an optimality criterion, we mean a pre-ordering (i.e., a transitive reflexive relation) $\preceq$ on the set $\mathcal{A}$.*
- *An optimality criterion is called G-invariant if for all $g \in G$, and for all $A, B \in \mathcal{A}$, $A \preceq B$ implies $g(A) \preceq g(B)$.*
- *An optimality criterion is called final if there exists one and only one element $A \in \mathcal{A}$ that is preferable to all the others, i.e., for which $B \preceq A$ for all $B \neq A$.*
- *An optimality criterion is called natural if it is $G-$invariant and final.*

Theorem. [2, 3] *Let M be a manifold, let G be a $d-$dimensional Lie transformation group on M, and let $\preceq$ be a natural (i.e., $G-$invariant and final) optimality criterion on the class $\mathcal{A}$ of all $r-$parametric families of sets from M, $r < d$. Then:*

- *the optimal family $A_{\rm opt}$ is $G-$invariant; and*
- *each set X from the optimal family is a union of orbits of $\geq (d-r)-$dimensional subgroups of the group G.*

(For readers' convenience, the proof is given in the end of the paper.)

Optimal families of sets for acupuncture and other techniques of traditional oriental medicine: first approximation. In applications to traditional oriental medicine, we are interested in sets $X \subset R^2$. As have already mentioned, for such sets, the natural group of symmetries G_a is generated by shifts and rotations. So, to apply our main result to these sets, we must describe all orbits of subgroups of G_a.

Since we are interested in sets which are different from the entire plane, we must look for 1-D orbits. A 1-D orbit is an orbit of a 1-D subgroup. This subgroup is uniquely determined by its "infinitesimal" element, i.e., by the corresponding element of the Lie algebra of the group G. This Lie algebra if easy to describe. For each of its elements, the corresponding differential equation (that describes the orbit) is reasonably easy to solve.

In geometric terms: it is known that each composition of shifts and rotations (i.e., in geometric terms, a *motion*) is either a *shift*, or a *rotation* around some point. Similarly, an infinitesimal composition is either an infinitesimal shift, or an infinitesimal rotation.

- If the infinitesimal element of a group is an infinitesimal *shift*, then the resulting group consists of all shifts in the corresponding direction, and the resulting orbit is a *straight line*.
- If the infinitesimal element of the group is an is an infinitesimal *rotation* around some point, then the resulting group consists of all rotations around this point, and the resulting orbit is a *circle*.

So, in the first approximation, we come to a conclusion that an optimal family of sets consists of either *straight lines* or *circles*. In other words, we conclude that all activation points must be located along one or several *lines* (straight or circular).

First conclusion. *This conclusion justifies the empirical fact that best activation points are indeed located around several lines called meridians.*

Optimal families of sets for acupuncture and other techniques of traditional oriental medicine: second approximation. That we must use points along a line is a good information, but we would like to be more specific than that and find out which points on the line we should use. In other words, it is desirable to move from a (too *general*) *first* approximation to a (more *specific*) *second* approximation.

In other words, we want to restrict ourselves, in every set from the original family, to a *subset*. According to the above result, every set from an optimal family consists of orbits of subgroups of the original symmetry group. Thus:

- Each first-approximation set is an orbit of a subgroup $G_0 \subseteq G_a$.
- Similarly, the desired *subset* of the original G_0-orbit set, must also be an orbit – an orbit of a *subgroup* $G_1 \subset G_0$ of the group G_0.

The group G_0 is a 1-D group – it is either the group of all rotations around a point, or the group of all shifts in a given direction. In both cases, all closed subgroups of such a group are known:

- For *rotations*, each closed subgroup consists of rotations by the angles 0, $2\pi/n$, $4\pi/n$, ..., $2(n-1)\pi/n$, where n is a positive integer.
- For *shifts*, i.e., for the case when G_0 is the group of all shifts by $\lambda \cdot \vec{e}$, where $\vec{e}$ is a fixed unit vector, and λ is an arbitrary real number, each closed subgroup $G_1 \subseteq G_0$ consists of shifts by $k \cdot \vec{e}_0$, where k is an arbitrary integer, and $\vec{e}_0 = \lambda_0 \cdot \vec{e}$ is a fixed vector in the direction of $\vec{e}$.

In both cases, the new orbit consists of *equidistant* points on the original line (i.e., on a straight line or on a circle).

Second conclusion. *This conclusion is also in good accordance with the experimental data about acupuncture points, most of which are located along the meridians at approximately the same distance from each other.*

Final conclusion. Thus, *the main geometry of acupuncture is theoretically justified.*

A similar geometric formalism is used to describe:

- shapes of *celestial objects* [2, 3, 4];
- shapes in *fracture theory*: for a symmetric body, each fault (crack, etc.) is a spontaneous symmetry violation [11]; this fact not only *explains* the shapes of the faults [11], it enables us to describe the best sensor locations for *detecting* these faults [8, 9, 10], etc.

(see also [7]). A general *symmetry* approach, with possibly non-geometric symmetries, enables us to explain the empirical optimality of different fuzzy, neural, genetic, and other techniques [6]. So, we hope that this approach will lead us even deeper into the foundations of traditional oriental medicine.

Proof of the Theorem. Since the criterion $\preceq$ is final, there exists one and only one optimal family of sets. Let us denote this family by $A_{\rm opt}$.

1. Let us first show that this family $A_{\rm opt}$ is indeed $G-$invariant, i.e., that $g(A_{\rm opt}) = A_{\rm opt}$ for every transformation $g \in G$.

Indeed, let $g \in G$. From the optimality of $A_{\rm opt}$, we conclude that for every $B \in \mathcal{A}$, $g^{-1}(B) \preceq A_{\rm opt}$. From the $G-$invariance of the optimality criterion, we can now conclude that $B \preceq g(A_{\rm opt})$. This is true for all $B \in \mathcal{A}$ and therefore, the family $g(A_{\rm opt})$ is optimal. But since the criterion is final, there is only one optimal family; hence, $g(A_{\rm opt}) = A_{\rm opt}$. So, $A_{\rm opt}$ is indeed invariant.

2. Let us now show an arbitrary set X_0 from the optimal family $A_{\rm opt}$ consists of orbits of $\geq (d - r)-$dimensional subgroups of the group G.

Indeed, the fact that $A_{\rm opt}$ is $G-$invariant means, in particular, that for every $g \in G$, the set $g(X_0)$ also belongs to $A_{\rm opt}$. Thus, we have a (smooth) mapping $g \to g(X_0)$ from the $d-$dimensional manifold G into the $\leq r-$dimensional set $G(X_0) = \{g(X_0) \,|\, g \in G\} \subseteq A_{\rm opt}$. In the following, we will denote this mapping by g_0.

Since $r < d$, this mapping cannot be 1-1, i.e., for some sets $X = g'(X_0) \in G(X_0)$, the pre-image $g_0^{-1}(X) = \{g \,|\, g(X_0) = g'(X_0)\}$ consists of one than one point. By definition of $g(X)$, we can conclude that $g(X_0) = g'(X_0)$ if and only if $(g')^{-1}g(X_0) = X_0$. Thus, this pre-image is equal to $\{g \,|\, (g')^{-1}g(X_0) = X_0\}$. If we denote $(g')^{-1}g$ by $\tilde{g}$, we conclude that $g = g'\tilde{g}$ and that the pre-image $g_0^{-1}(X) = g_0^{-1}(g'(X_0))$ is equal to $\{g'\tilde{g} \,|\, \tilde{g}(X_0) = X_0\}$, i.e., to the result of applying g' to $\{\tilde{g} \,|\, \tilde{g}(X_0) = X_0\} = g_0^{-1}(X_0)$. Thus, each pre-image $(g_0^{-1}(X) = g_0^{-1}(g'(X_0)))$ can be obtained from one of these pre-images (namely, from $g_0^{-1}(X_0)$) by a smooth invertible transformation g'. Thus, all pre-images have the same dimension D.

We thus have a *stratification* (fiber bundle) of a $d-$dimensional manifold G into $D-$dimensional strata, with the dimension D_f of the factor-space being $\leq r$. Thus, $d = D + D_f$, and from $D_f \leq r$, we conclude that $D = d - D_f \geq n - r$.

So, for every set $X_0 \in A_{\rm opt}$, we have a $D \geq (n - r)-$dimensional subset $G_0 \subseteq G$ that leaves X_0 invariant (i.e., for which $g(X_0) = X_0$ for all $g \in G_0$). It is easy to check that if $g, g' \in G_0$, then $gg' \in G_0$ and $g^{-1} \in G_0$, i.e., that G_0 is a *subgroup* of the group G. From the definition of G_0 as $\{g \mid g(X_0) = X_0\}$ and the fact that $g(X_0)$ is defined by a smooth transformation, we conclude that G_0 is a smooth sub-manifold of G, i.e., a $\geq (n-r)-$dimensional subgroup of G.

To complete our proof, we must show that the set X_0 is a union of orbits of the group G_0. Indeed, the fact that $g(X_0) = X_0$ means that for every $x \in X_0$, and for every $g \in G_0$, the element $g(x)$ also belongs to X_0. Thus, for every element x of the set X_0, its entire orbit $\{g(x) \mid g \in G_0\}$ is contained in X_0. Thus, X_0 is indeed the union of orbits of G_0. The theorem is proven.

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