

Interval Computations, Soft Computing, and Aerospace Applications

(Research Report)

Vladik Kreinovich

NASA Pan-American Center for
Earth and Environmental Studies (PACES)
The University of Texas at El Paso
El Paso, TX 79968, USA
email vladik@cs.utep.edu

1 Introduction: Data Processing and Interval Computations

Data processing. In many real-life problems, we are interested in the value y of a physical quantity which is *difficult* or *impossible* to measure directly.

For example, we cannot directly measure the distance to a star, or the amount of oil in a given area.

To measure this quantity y , we:

- measure some other quantities x_1, \dots, x_n which are related to y by a known dependence $y = f(x_1, \dots, x_n)$, and then
- compute the estimate \tilde{y} for the desired quantity y by applying the algorithm f to the results \tilde{x}_i of measuring the quantities x_i : $\tilde{y} = f(\tilde{x}_1, \dots, \tilde{x}_n)$.

This two-stage process is called *indirect measurement*, and computing f is called *data processing*.

For example, to estimate the amount of oil in a given area, we may use geophysical data plus satellite images of this area.

Error estimation of the results of data processing: mathematical statistics and interval computations. Values \tilde{x}_i come from measurements, and measurements are never 100% accurate; therefore, $\tilde{x}_i \neq x_i$. Due to the inaccuracies $\Delta x_i = \tilde{x}_i - x_i$ of direct measurements, the result $\tilde{y} = f(\tilde{x}_1, \dots, \tilde{x}_n)$ is, in general, different from the desired value $y = f(x_1, \dots, x_n)$: $\Delta y = \tilde{y} - y \neq 0$. In practical applications, it is extremely important to know what are the possible values of the difference Δy .

For example, if our estimate for amount of oil in a given area is ≈ 100 mln. ton, then whether we start exploiting this oil or not depends on the accuracy of this estimate:

- *If the measurement error Δy does not exceed 10 mln. ton, then the actual value can be anywhere from 90 to 100, and we should recommend exploitation.*
- *On the other hand, if the measurement error Δy can be as large as 100 mln. ton, then this means that the actual value y can actually be equal to 0 (meaning that there may be no oil at all). In this case, further, more accurate measurements are needed because we can make a decision.*

To estimate Δy , we must have some information about the errors Δx_i of direct measurements. What type of information can we have?

- The manufacturer of the measuring instrument gives us a *guaranteed* error Δ_i , i.e., a value for which $|\Delta x_i| \leq \Delta_i$.

Without such a guarantee, a measurement result does not restrict possible values of x_i and thus, it is not a measurement.

- In some cases, in addition to the upper bounds Δ_i , we know *probabilities* of different values of Δx_i .

If we know probabilities, then we have a typical problem of *mathematical statistics*: given probability distributions for $\Delta x_i = \tilde{x}_i - x_i$, find the probability distribution for $y = f(x_1, \dots, x_n)$. To get the probabilities of Δx_i , we *calibrate* the measuring instrument, i.e., we compare its results with the results of a better (standard) measuring instrument.

An application of statistical methods to environmentally-oriented multispectral satellite image processing is given in [29].

However, there are two important situations when we do not know these probabilities:

- In *fundamental physics*, we perform measurements on the *cutting edge*, so no better instrument is possible at all.

- In *manufacturing*, calibration of all sensors is potentially possible, but, in practice, too expensive.

When we do not know the probabilities, we only know that $|\tilde{x}_i - x_i| \leq \Delta_i$, i.e., the only information about x_i is that x_i belongs to the *interval* $[\tilde{x}_i - \Delta_i, \tilde{x}_i + \Delta_i]$.

For example, if the measured value of the current is $\tilde{x} = 1$ A, and the manufacturer guarantees the measurement error to be within ± 0.1 A, then the actual value of x can be any number from the interval $[0.9, 1.1]$.

In this case, the problem of estimating the error of indirect measurement can be reformulated as follows:

- we know n intervals $\mathbf{x}_i = [\tilde{x}_i - \Delta_i, \tilde{x}_i + \Delta_i]$,
- we know an algorithm f which transforms n real numbers x_1, \dots, x_n into a real number y , and
- we want to compute the interval

$$\mathbf{y} = f(\mathbf{x}_1, \dots, \mathbf{x}_n) = \{f(x_1, \dots, x_n) \mid x_i \in \mathbf{x}_i\}.$$

This problem is called the basic problem of *interval computations*.

Linearization is not always possible. If a function f is smooth, and the errors Δx_i are small, then we can neglect quadratic terms in f , and get explicit formulas for \mathbf{y} . Due to our approximation, we get *approximate* endpoints of the interval \mathbf{y} : the actual values y can be, therefore, slightly outside this approximate interval.

In many applications, it is OK, but in some real-life situations, the consequences of a possible error are so serious that we need to *guarantee* that y is contained in the resulting interval \mathbf{y} . An example of this problem is planning a mission to the Moon. To get guaranteed estimates for this problem, Ramon E. Moore, then Stanford's Ph.D. student working on 1959 NASA-oriented project, designed new techniques called *interval computations*.

2 Interval Computations in Aerospace Applications: Why

Let us enumerate the reasons why methods of interval computations are needed in aerospace applications:

- First, we want to *guarantee* a mission, we want to *guarantee* that a spaceship hits the Moon (or another planet), and interval computations provide us with the *guaranteed* computation results.

- Second, according to the new NASA paradigm, we need all the missions to be *faster, better, cheaper*. This means, in particular, that we should preferably use off-shelf components, with no time to individually calibrate all of them (and thus, no time to find all the probabilities).
- Third, many NASA missions are missions into the unknown. We simply do not know the exact values of the parameters characterizing the distant planet's surface, or the corresponding probabilities; the only thing we may know for planning a mission are *intervals* of possible values of these parameters.
- Finally, one of the main goals of NASA missions is to produce *solid scientific results*, and “solid” means *guaranteed*.

3 Aerospace Applications of Interval Computations: Examples

Robot navigation. A mobile robot has to navigate in an unknown environment by using imprecise sensors. Traditionally, statistical approach was used to describe the sensor's uncertainty, but this approach has two main drawbacks: it is very costly to calibrate, and it cannot be applied in an unknown environment, when we have no time to calibrate first. To avoid these problems, we used *interval uncertainty* in a UTEP robot. This robot won 1st place in the international competition at AAAI'97: it was more efficient, less error-prone, and at the same time rather simple to program. This technique can be used in future planetary missions.

Telemanipulation [42]. The idea of telemanipulation, when a robotic arm repeats the movements of the operator's arm, works perfectly well in the movies, but not so perfectly well in the real space exploration. The reasons for this imperfection are simple: both sensors (which measure the operator's movements) and the actuators (which copy them) are inaccurate. The more complicated the robotic arm, the more actuators it uses, and the more inaccuracy accumulates. It turns out that if we take interval inaccuracy into consideration, we can greatly improve the performance of the telemanipulator – namely, of the state-of-the-art MIT/Utah robotic arm.

Multi-spectral satellite imaging [30.58]. The existing Earth-imaging satellites of Landsat series, whose ability is restricted to 7 channels only, already send Gigabytes of difficult-to-process information. For some imaging problems, 7 channels are not sufficient, so new satellites will be able to scan 500 channels. With 100 times more data, we need at least 100 times more time to process it; even now, processing all the satellite data is a problem, and with the expected two orders of magnitude increase, this processing seems to be getting close to impossible. Solution: take interval uncertainty into consideration. It turns out

that with this uncertainty in mind, we can use *linear* models where previously only complex models were used; computations become *faster* and thus, quite feasible.

Non-destructive testing of aerospace constructions [22,63]. Failure of an aerospace apparatus can be disastrous, and therefore, all mechanical parts must be thoroughly tested. Exhaustive testing, however, is extremely expensive. Here also intervals help. It turns out that:

- when the tested surface is smooth (no faults, no cracks, etc.), the dependence of the measured signal on the test ultrasound signal is also smooth, and since the test signals are small, we can approximate it by a linear dependence;
- on the other hand, if there are non-smoothnesses (faults, cracks, etc.), then non-linear terms are no longer negligible.

Checking whether the actual data is consistent with the linear dependence (within interval uncertainty), we can thus test whether there is a non-smoothness. Experiments confirmed that this is a viable and expense-saving testing method.

We also analyzed the problem of choosing the best sensor locations for aerospace testing [26,55,56].

Geophysical tomography [4]. Interval computations help in reconstructing the geophysical structure from observations.

Energy from space: a possible future application of interval computations. Solar energy is a very prospective renewable energy resource, but on-Earth Solar stations are not perfect: they occupy large pieces of land, they do not work in bad weather, etc. An ideal solution would be to use *orbital* solar power stations, which would generate electricity and then transmit it to Earth as a microwave beam. The problem with this solution is that a high-energy microwave beam may damage whatever it accidentally hits. So, the better solution is to have several orbital stations and several receivers, so that the resulting beams do not reach the dangerous level. Again, interval methods provide a solution to this problem.

4 Related Research: Feasible Algorithms and Impossibility Results

First specific problem: space is unreliable [2]. When designing algorithms for space applications, we face a specific problem: space is unreliable; a computer may stop before finishing computations. It is therefore desirable to have algorithms which produce some (approximate) results when interrupted. It turns out that for guaranteed (interval) algorithms, it is theoretically possible

to transform each algorithm into an interruptible one without greatly increasing its computation time. This is still a rather theoretical result, with few practical examples.

Second specific problem: reusing software [36]. A huge portion of a space mission's cost consists in designing software. A natural way of saving this costs is to *reuse* the software which was already produced for other missions (or for similar computational problems). Therefore, it seems natural to design new software in such a way that this software be used not only for this particular mission, but for similar future missions as well. The necessity to take the future use into consideration adds cost to writing new software. Hence, if we promote reuse:

- on one hand, we *save costs* on reusing software components, but
- on the other hand, we *add costs* to make new software components reusable.

It is, therefore, not clear whether a reuse policy will actually save costs or not. In [36], we show how the use of interval uncertainty can help in answering this question.

General research in interval computations. Due to the importance of interval computations in aerospace applications, we have researched the possibility of designing feasible algorithms for solving various interval computation problems.

- The general *analysis* is given in [1] (for linear systems).
- Feasible algorithms are produced:
 - in [23,49] for error estimation for linearized indirect measurements;
 - in [7,8] for function approximation;
 - in [13,37] for optimization.

In most of these cases, we produced the *optimal* algorithms based on the general group-theoretic approach (borrowed from physics [62]).

- In some cases, we showed that the corresponding interval problem cannot, in the general case, be feasibly solved; these results cover, in particular:
 - solving systems of interval linear equations [6];
 - optimal function approximation [7,8], and
 - signal processing [9].

In some cases, it is clear whether an algorithm is feasible or not, but in some borderline cases, checking feasibility requires a complicated theoretical analysis [18,20].

All major results have been summarized in our monograph [10]; aerospace applications are surveyed in [47].

Comment. Some of these results also have non-aerospace applications, e.g., to medicine [22,31,60,63].

5 From Interval Computations to Soft Computing

Why soft computing. As we have mentioned, some interval computation problems are not feasible; this means that if we do not have any additional information, we cannot, in general, solve these problems efficiently. We can rephrase this negative result in a positive form: to solve these problems, we must add some *expert knowledge*. The methodologies which use expert knowledge to solve problems are known as *soft computing*; so, we can reformulate our conclusion as saying that many aerospace problems require soft computing.

We have shown that the use of soft computing methods can indeed make these problems feasibly solvable [34].

Two main problems of satellite data processing. One of the main objectives of PACES is processing satellite data with the purpose of extracting useful geophysical, environmental, and other earth-related information. For this data processing to be successful, we need to solve two major problems:

- First, satellite imaging provides us with an unusually *enormous* amount of data; traditional methods of data processing, which work well for smaller amounts of data, often require too long a time when applied to satellite images; thus, new methods are needed.
- Second, many traditional data and image processing techniques depend on *experts to do many routine subtasks* such as mosaicking images, identifying different vegetation or cloud patterns, etc. With a huge amount of data coming from the satellites, it is no longer possible to use experts to process all this data, these subtasks need to be automated.

In solving both problems, soft computing techniques such as fuzzy, neural, etc., naturally emerge.

Soft computing helps in solving the first problem of satellite data processing.

- Traditional methods of data processing are based on thorough statistical analysis of the problems.

- Due to the continuing progress in satellite imaging techniques and to the continuing discovery of new applications, there is no time to follow a (rather slow) traditional statistical analysis approach. Therefore, new heuristic methods are needed, methods which use, in addition to statistics, also informal expert ideas.

Fuzzy, neural, and other soft computing techniques allow us:

- to *formalize* these expert ideas, and
- which is very important, to formalize these ideas in a scientifically justified consistent fashion, thus *increasing* the *reliability* of the results of data processing.

Examples of such formalizations are given in [16,28,32,39,40]. An important heuristic idea is the idea of choosing the *simplest* explanation. In computer science, there are natural measures of complexity and simplicity, such as the length and the time of the program, but with respect to all these formal measures, finding the simplest explanation becomes a computationally un-feasible task; soft computing enables us to explain the existing feasible modifications of this idea and to come up with alternative feasible modifications [11,21,24,33,44].

These explanations help not only in heuristic image processing and data processing, but also:

- in education [43],
- in decision making [61],
- in humanities [25], etc.

Soft computing helps in solving the first problem of satellite data processing.

- Experts have trouble describing how exactly they mosaic or how exactly they identify features.
- Experts can, at best, formulate their rules in terms of words of natural language (like “a little bit”). To us these informal rules, we must use a special techniques for transforming such rules into automated control: fuzzy logic.
- If even rules are not available, then the only way to automate is to observe the experts’ behavior in several cases and extrapolate. One of the best extrapolation techniques, which is the most appropriate for our purposes because it simulates the way humans do extrapolation, is neural networks.

Applications of soft computing methodology include *image processing* (including processing satellite images and clustering) [27,45,46], as well as related problems such as:

- optimization [15];
- control [14,51,53]; and
- modeling [12].

A general survey of soft computing methodology is given in [52].

In many real-life situations, the existing soft computing techniques are still too computationally intensive [50]; in the attempts to solve this problem, the following direction were pursued:

- thorough analysis of the modifications of soft computing methodologies which have already been proposed but which have not yet been practically used, with the hope that some of these modifications will help to make our problems computationally feasible [17];
- designing new (e.g., multi-D or hierarchical) modifications of soft computing methodologies [19,41,48,52,57,59], with the hope that these new methodologies will lead to feasible solutions to the problems;
- combining soft computing methods with alternative computationally feasible techniques for processing uncertainty, such as *logic programming* [16,54];
- analyzing the possibility of using new physical and engineering ideas in computer design [38].

References

Refereed papers, books, and book chapters (published)

1. G. Alefeld, V. Kreinovich, and G. Mayer, "The Shape of the Solution Set for Systems of Interval Linear Equations with Dependent Coefficients", *Mathematische Nachrichten*, 1998, Vol. 192, pp. 23–36.
2. M. Beltran, G. Castillo, and V. Kreinovich, "Algorithms That Still Produce a Solution (Maybe Not Optimal) Even When Interrupted: Shary's Idea Justified", *Reliable Computing*, 1998, Vol. 4, No. 1, pp. 39–53.
3. D. Dennis, V. Kreinovich, and S. Rump, "Intervals and the Origin of Calculus", *Reliable Computing*, 1998, Vol. 4, No. 2, pp. 191–197.
4. D. I. Doser, K. D. Crain, M. R. Baker, V. Kreinovich, and M. C. Gerstenberger, "Estimating uncertainties for geophysical tomography", *Reliable Computing*, 1998, Vol. 4, No. 3, pp. 241–268.
5. A. Q. Gates and V. Kreinovich, "Why is a function defined as a set of ordered pairs?", *ACM SIGSCE Bulletin*, 1997, Vol. 29, No. 4, p. 57.

6. G. Heindl, V. Kreinovich, and A. V. Lakeyev, "Solving Linear Interval Systems is NP-Hard Even If We Exclude Overflow and Underflow", *Reliable Computing*, 1998, Vol. 4, No. 4, pp. 383–388.
7. M. Koshelev and L. Longpré, "Approximation of Interval Functions", Chapter 19 in: V. Kreinovich, A. Lakeyev, J. Rohn, and P. Kahl, *Computational complexity and feasibility of data processing and interval computations*, Kluwer, Dordrecht, 1997, pp. 207–217.
8. M. Koshelev, L. Longpré, and P. Taillibert, "Optimal Approximation of Quadratic Interval Functions", *Reliable Computing*, 1998, Vol. 4, No. 4, pp. 351–360.
9. O. Kosheleva *et al.*, "Engineering corollary: signal processing is NP-hard", Chapter 14 in: V. Kreinovich, A. Lakeyev, J. Rohn, and P. Kahl, *Computational complexity and feasibility of data processing and interval computations*, Kluwer, Dordrecht, 1997, pp. 153–158.
10. V. Kreinovich, A. Lakeyev, J. Rohn, and P. Kahl, *Computational complexity and feasibility of data processing and interval computations*, Kluwer, Dordrecht, 1997.
11. V. Kreinovich and L. Longpré, "Human Visual Perception and Kolmogorov Complexity: Revisited", *Bulletin of the European Association for Theoretical Computer Science (EATCS)*, 1998, Vol. 64, pp. 155–158.
12. V. Kreinovich, G. C. Mouzouris, and H. T. Nguyen, "Fuzzy rule based modeling as a universal approximation tool", In: H. T. Nguyen and M. Sugeno (eds.), *Fuzzy Systems: Modeling and Control*, Kluwer, Boston, MA, 1998, pp. 135–195.
13. V. Kreinovich, S. Starks, and G. Mayer, "On a Theoretical Justification of The Choice of Epsilon-Inflation in PASCAL-XSC", *Reliable Computing*, 1997, Vol. 3, No. 4, pp. 437–452.
14. H. T. Nguyen and V. Kreinovich, "Methodology of fuzzy control: an introduction", In: H. T. Nguyen and M. Sugeno (eds.), *Fuzzy Systems: Modeling and Control*, Kluwer, Boston, MA, 1998, pp. 19–62.
15. H. T. Nguyen and V. Kreinovich, "Multi-criteria optimization – an important foundation of fuzzy system design", In: L. Reznik, V. Dimitrov, and J. Kacprzyk, *Fuzzy system design: social and engineering applications*, Physica Verlag, Heidelberg, 1998, pp. 24–35.
16. H. T. Nguyen and V. Kreinovich, "Using Gelfond-Przymusinska's Epistemic Specifications to Justify (Some) Heuristic Methods Used in Expert Systems and Intelligent Control", *Soft Computing*, 1997, Vol. 1, No. 4, pp. 198–209.

17. H. T. Nguyen, V. Kreinovich, and P. Wojciechowski, "Strict Archimedean t-Norms and t-Conorms as Universal Approximators", *International Journal of Approximate Reasoning*, 1998, Vol. 18, pp. 239–249.

Refereed papers and book chapters (accepted)

18. A. Blass, Y. Gurevich, V. Kreinovich, and L. Longpré, "A Variation on the Zero-One Law", *Information Processing Letters*, 1998 (to appear).
19. B. Cloteaux, C. Eick, B. Bouchon-Meunier, and V. Kreinovich, "From Ordered Beliefs to Numbers: How to Elicit Numbers Without Asking for Them (Doable but Computationally Difficult)", *International Journal of Intelligent Systems*, 1998 (to appear).
20. D. E. Cooke, V. Kreinovich, and L. Longpré, "Which algorithms are feasible? MaxEnt approach", In: G. Erickson (ed.), *Maximum Entropy and Bayesian Methods*, Kluwer, Dordrecht, 1998 (to appear).
21. D. Fox, M. Schmidt, M. Koshelev, V. Kreinovich, L. Longpré, and J. Kuhn, "We must choose the simplest physical theory: Levin-Li-Vitányi theorem and its potential physical applications", In: G. Erickson (ed.), *Maximum Entropy and Bayesian Methods*, Kluwer, Dordrecht, 1998 (to appear).
22. O. Kosheleva, S. Cabrera, R. Osegueda, S. Nazarian, D. L. George, M. J. George, V. Kreinovich, and K. Worden, "Case study of nonlinear inverse problems: mammography and non-destructive evaluation", In: A. Mohamad-Djafari (ed.), *Bayesian Inference for Inverse Problems*, Proceedings of the SPIE/International Society for Optical Engineering, Vol. 3459, San Diego, CA, 1998 (to appear).
23. V. Kreinovich, "A simplified version of the tomography problem can help to estimate the errors of indirect measurements", In: A. Mohamad-Djafari (ed.), *Bayesian Inference for Inverse Problems*, Proceedings of the SPIE/International Society for Optical Engineering, Vol. 3459, San Diego, CA, 1998 (to appear).
24. V. Kreinovich, L. Longpré, and M. Koshelev, "Kolmogorov complexity, statistical regularization of inverse problems, and Birkhoff's formalization of beauty", In: A. Mohamad-Djafari (ed.), *Bayesian Inference for Inverse Problems*, Proceedings of the SPIE/International Society for Optical Engineering, Vol. 3459, San Diego, CA, 1998 (to appear).
25. H. T. Nguyen, V. Kreinovich, and V. Shekhter, "On the Possibility of Using Complex Values in Fuzzy Logic For Representing Inconsistencies", *International Journal of Intelligent Systems*, 1998, Vol. 13, No. 8, pp. 683–714.

26. R. Osegueda, C. Ferregut, M. J. George, J. M. Gutierrez, and V. Kreinovich, "Maximum entropy approach to optimal sensor placement for aerospace non-destructive testing", In: G. Erickson (ed.), *Maximum Entropy and Bayesian Methods*, Kluwer, Dordrecht, 1998 (to appear).
27. A. T. Popov, H. T. Nguyen, and L. K. Reznik, "An Application of Fuzzy Mathematical Morphology to Interval-Valued Knowledge Representation: A Remark", *Reliable Computing* (to appear).
28. E. R. Scerri, V. Kreinovich, P. Wojciechowski, and R. R. Yager, "Ordinal Explanation of the Periodic System of Chemical Elements", *International Journal of Uncertainty, Fuzziness, and Knowledge-Based Systems (IJUFKS)* (to appear).
29. S. A. Starks and V. Kreinovich, "Environmentally-oriented processing of multi-spectral satellite images: new challenges for Bayesian methods", In: G. Erickson (ed.), *Maximum Entropy and Bayesian Methods*, Kluwer, Dordrecht, 1998 (to appear).
30. S. A. Starks and V. Kreinovich, "Multi-spectral inverse problems in satellite image processing", In: A. Mohamad-Djafari (ed.), *Bayesian Inference for Inverse Problems*, Proceedings of the SPIE/International Society for Optical Engineering, Vol. 3459, San Diego, CA, 1998 (to appear).

National and International conferences:

31. K. M. Aló, R. Aló, A. de Korvin, and V. Kreinovich, "Spinal Cord Stimulation for Chronic Pain Management: Towards an Expert System", *Proceedings of the 4th World Congress on Expert Systems, Mexico City, March 16-20, 1998*, Vol. 1, pp. 156-164.
32. D. E. Cooke, V. Kreinovich, and S. A. Starks, "ALPS: A Logic for Program Synthesis (Motivated by Fuzzy Logic)", *Proceedings of the FUZZ-IEEE'98 International Conference on Fuzzy Systems*, Anchorage, Alaska, May 4-9, 1998, Vol. 1, pp. 779-784.
33. F. Fernandez, V. Kreinovich, and L. Longpré, "Justification of Rissanen's Approximate Formula for Prior Probability", *Complexity Conference Abstracts 1998*, June 1998, Abstract No. 98-11, p. 13.
34. O. N. Garcia, V. Kreinovich, L. Longpré, and H. T. Nguyen, "Complex problems: granularity is necessary, granularity helps", In: H. P. Nguyen (ed.), *Proceedings of the Vietnam-Japan International Symposium on Fuzzy Systems and Applications VJFUZZY'98*, September 30-October 2, 1998, Ha Long Bay, Vietnam (to appear).

36. A. Gates, V. Kreinovich, L. Sifuentes, and S. Starks, "OO Or Not OO: When Object-Oriented is Better. Qualitative Analysis and Application to Satellite Image Processing", In: G. Alefeld and R. A. Trejo (eds.), *Interval Computations and its Applications to Reasoning Under Uncertainty, Knowledge Representation, and Control Theory. Proceedings of MEXICON'98, Workshop on Interval Computations, 4th World Congress on Expert Systems*, México City, México, 1998.
37. R. B. Kearfott and V. Kreinovich, "Where to Bisect a Box? A Theoretical Explanation of the Experimental Results", In: G. Alefeld and R. A. Trejo (eds.), *Interval Computations and its Applications to Reasoning Under Uncertainty, Knowledge Representation, and Control Theory. Proceedings of MEXICON'98, Workshop on Interval Computations, 4th World Congress on Expert Systems*, México City, México, 1998.
38. M. Koshelev and V. Kreinovich, "Towards Computers of Generation Omega – Non-Equilibrium Thermodynamics, Granularity, and Acausal Processes: A Brief Survey", *Proceedings of the International Conference on Intelligent Systems and Semiotics (ISAS'97)*, National Institute of Standards and Technology Publ., Gaithersburg, MD, 1997, pp. 383–388.
39. M. Koshelev, V. Kreinovich, H. T. Nguyen, and B. Bouchon-Meunier, "Uncertainty representation explains and helps methodology of physics and science in general", In: H. P. Nguyen (ed.), *Proceedings of the Vietnam-Japan International Symposium on Fuzzy Systems and Applications VJFUZZY'98*, September 30–October 2, 1998, Ha Long Bay, Vietnam (to appear).
40. O. Kosheleva, V. Kreinovich, B. Bouchon-Meunier, and R. Mesiar, "Operations with Fuzzy Numbers Explain Heuristic Methods in Image Processing", *Proceedings of the International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU'98)*, Paris, France, July 6–10, 1998.
41. O. Kosheleva, V. Kreinovich, H. T. Nguyen, and B. Bouchon-Meunier, "How to describe partially ordered preferences: mathematical foundations", In: H. P. Nguyen (ed.), *Proceedings of the Vietnam-Japan International Symposium on Fuzzy Systems and Applications VJFUZZY'98*, September 30–October 2, 1998, Ha Long Bay, Vietnam (to appear).
42. V. Kreinovich and L. O. Fuentes, "Telemanipulation: The Virtual Tool Approach and Its Interval-Based Justification", In: G. Alefeld and R. A. Trejo (eds.), *Interval Computations and its Applications to Reasoning Under Uncertainty, Knowledge Representation, and Control Theory. Proceedings of MEXICON'98, Workshop on Interval Computations, 4th World Congress on Expert Systems*, México City, México, 1998.

43. V. Kreinovich, E. Johnson-Holubec, L. K. Reznik, and M. Koshelev, "Co-operative learning is better: explanation using dynamical systems, fuzzy logic, and geometric symmetries", In: H. P. Nguyen (ed.), *Proceedings of the Vietnam-Japan International Symposium on Fuzzy Systems and Applications VJFUZZY'98*, September 30–October 2, 1998, Ha Long Bay, Vietnam (to appear).
44. V. Kreinovich, L. Longpré, and H. T. Nguyen, "Towards formalization of feasibility, randomness, and commonsense implication: Kolmogorov complexity, and the necessity of considering (fuzzy) degrees", In: H. P. Nguyen (ed.), *Proceedings of the Vietnam-Japan International Symposium on Fuzzy Systems and Applications VJFUZZY'98*, September 30–October 2, 1998, Ha Long Bay, Vietnam (to appear).
45. V. Kreinovich, H. T. Nguyen, S. A. Starks, and Y. Yam, "Decision making based on satellite images: optimal fuzzy clustering approach", *Proceedings of the 37th IEEE Conference on Decision and Control CDC'98*, Tampa, Florida, December 16–18, 1998 (to appear).
46. V. Kreinovich, H. T. Nguyen, and Y. Yam, "Optimal Choices of Potential Functions in Fuzzy Clustering", *The Chinese University of Hong Kong, Department of Mechanical & Automation Engineering*, Technical Report CUHK-MAE-98-001, January 1998.
47. V. Kreinovich and S. A. Starks, "Applications of Interval Computations: an Overview with a Special Emphasis on Actual and Potential Aerospace Applications", *Abstracts of the SIAM Annual Meeting*, Toronto, July 13–17, 1998, pp. 34–35.
48. V. Kreinovich and S. A. Starks, "A new 5D geometric formalism for physics and for data processing", *Abstracts of American Mathematical Society*, 1997, Vol. 18, No. 3, p. 460.
49. V. Kreinovich, S. A. Starks, and R. Trejo, "Automatic Differentiation or Monte-Carlo Methods: Which is Better for Error Estimation?", *Abstracts of the SIAM Annual Meeting*, Toronto, July 13–17, 1998, p. 51.
50. H. T. Nguyen, M. Koshelev, O. Kosheleva, V. Kreinovich, and R. Mesiar, "Computational Complexity and Feasibility of Fuzzy Data Processing: Why Fuzzy Numbers, Which Fuzzy Numbers, Which Operations with Fuzzy Numbers", *Proceedings of the International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU'98)*, Paris, France, July 6–10, 1998.

51. H. T. Nguyen and V. Kreinovich, "A Modification of Sugeno Integral Describes Stability and Smoothness of Fuzzy Control", *Proceedings of the FUZZ-IEEE'98 International Conference on Fuzzy Systems*, Anchorage, Alaska, May 4–9, 1998, Vol. 1, pp. 360–365.
52. H. T. Nguyen and V. Kreinovich, "Possible new directions in mathematical foundations of fuzzy technology: a contribution to the mathematics of fuzzy theory", In: H. P. Nguyen (ed.), *Proceedings of the Vietnam-Japan International Symposium on Fuzzy Systems and Applications VJ-FUZZY'98*, September 30–October 2, 1998, Ha Long Bay, Vietnam (to appear).

53. H. T. Nguyen, V. Kreinovich, and R. Aló, “Adding Fuzzy Integral to Fuzzy Control”, *Proceedings of the International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU'98)*, Paris, France, July 6–10, 1998.
54. H. T. Nguyen, V. Kreinovich, D. E. Cooke, Luqi, and O. Koshel'eva, “Towards combining fuzzy and logic programming techniques”, In: H. P. Nguyen (ed.), *Proceedings of the Vietnam-Japan International Symposium on Fuzzy Systems and Applications VJFUZZY'98*, September 30–October 2, 1998, Ha Long Bay, Vietnam (to appear).
55. R. Osegueda, C. Ferregut, M. J. George, J. M. Gutierrez, and V. Kreinovich, “Computational geometry and artificial neural networks: a hybrid approach to optimal sensor placement for aerospace NDE”, In: C. Ferregut, R. Osegueda, and A. Nuñez (eds.), *Proceedings of the International Workshop on Intelligent NDE Sciences for Aging and Futuristic Aircraft*, El Paso, TX, September 30–October 2, 1997, pp. 59–71.
56. R. Osegueda, C. Ferregut, M. J. George, J. M. Gutierrez, and V. Kreinovich, “Non-Equilibrium Thermodynamics Explains Semiotic Shapes: Applications to Astronomy and to Non-Destructive Testing of Aerospace Systems”, *Proceedings of the International Conference on Intelligent Systems and Semiotics (ISAS'97)*, National Institute of Standards and Technology Publ., Gaithersburg, MD, 1997, pp. 378–382.
57. S. A. Starks and V. Kreinovich, “Non-Interval Extension of Interval Methods Leads to a New 5D Geometric Formalism for Physics and Data Processing”, *International Conference on Interval Methods and their Application in Global Optimization (INTERVAL'98)*, April 20–23, Nanjing, China, Abstracts, 1998, pp. 136–138.
58. S. Starks and V. Kreinovich, “Soft Computing: Frontiers? A Case Study of Hyper-Spectral Satellite Imaging”, *Working Notes of the AAAI Symposium on Frontiers in Soft Computing and Decision Systems*, Boston, MA, November 8–10, 1997, p. 66–71.
59. S. A. Starks, V. Kreinovich, and A. Meystel, “Multi-Resolution Data Processing: It is Necessary, It is Possible, It is Fundamental”, *Proceedings of the International Conference on Intelligent Systems and Semiotics (ISAS'97)*, National Institute of Standards and Technology Publ., Gaithersburg, MD, 1997, pp. 145–150.

60. S. A. Starks, H. T. Nguyen, V. Kreinovich, H. P. Nguyen, and M. Navara, "Strong Negation: Its Relation to Intervals and Its Use in Expert Systems", In: G. Alefeld and R. A. Trejo (eds.), *Interval Computations and its Applications to Reasoning Under Uncertainty, Knowledge Representation, and Control Theory. Proceedings of MEXICON'98, Workshop on Interval Computations, 4th World Congress on Expert Systems*, México City, México, 1998.
61. R. Trejo and V. Kreinovich, "Complexity of Collective Decision Making Explained by Neural Network Universal Approximation Theorem", In: G. Alefeld and R. A. Trejo (eds.), *Interval Computations and its Applications to Reasoning Under Uncertainty, Knowledge Representation, and Control Theory. Proceedings of MEXICON'98, Workshop on Interval Computations, 4th World Congress on Expert Systems*, México City, México, 1998.
62. R. A. Trejo, V. Kreinovich, and L. Longpré, "Choosing a Physical Model: Why Symmetries?", *Complexity Conference Abstracts 1998*, June 1998, Abstract No. 98-37, p. 39.
63. K. Worden, R. Osegueda, C. Ferregut, S. Nazarian, E. Rodriguez, D. L. George, M. J. George, V. Kreinovich, O. Kosheleva, and S. Cabrera, "Interval Approach to Non-Destructive Testing of Aerospace Structures and to Mammography", In: G. Alefeld and R. A. Trejo (eds.), *Interval Computations and its Applications to Reasoning Under Uncertainty, Knowledge Representation, and Control Theory. Proceedings of MEXICON'98, Workshop on Interval Computations, 4th World Congress on Expert Systems*, México City, México, 1998.