Multi-Resolution Techniques in the Rules-Based Intelligent Control Systems: A Universal Approximation Result

Yeung Yam
Department of Mechanical
& Automation Engineering
The Chinese University of Hong Kong
Shatin, NT, Hong Kong, China
email yyam@mae.cuhk.edu.hk

Hung T. Nguyen
Department of Mathematical Sciences
New Mexico State University
Las Cruces, NM 88003, USA
email hunguyen@nmsu.edu

Vladik Kreinovich
Department of Computer Science
University of Texas at El Paso
El Paso, TX 79968, USA
email vladik@cs.utep.edu

Abstract

Intelligent control is a very successful method of transforming expert knowledge of control rules (formulated in terms of natural language, like "small") into a precise control strategy. It has led to many spectacular applications, ranging from appliances to automatic subway control to super-precise temperature control on a Space Shuttle mission.

It is known that fuzzy control is a *universal approximator*, i.e., that it can approximate every possible control strategy within an arbitrary accuracy. One of the main problems of fuzzy control is that the number of rules which are necessary to represent a given control strategy with a given accuracy, grows exponentially with the increase in accuracy. As a result, for reasonable accuracy, and a reasonable number of input variable, we sometimes need astronomically many rules.

In this paper, we start to solve this problem by pointing out that traditional one-step fuzzy rule bases, in which expert rules directly express control in terms of the input, are often a simplification of the actual multi-step expert reasoning. We show that a natural formalization of such expert reasoning leads to a universal approximation result in which the number of control rules does not increase with the increase in accuracy. Thus, this multi-resolution approach looks like a promising solution to the rule base explosion problem.

1 Intelligent Control: Rules Explosion as a Main Problem

Intelligent control is a very successful method of transforming expert knowledge of control rules (formulated in terms of natural language, like "small") into a precise control strategy. During the 25+ years of its existence, it has led to many spectacular applications, ranging from appliances to automatic subway control to super-precise temperature control on a Space Shuttle mission (see, e.g., [8, 9]).

The fact that intelligent control is a reasonable strategy is emphasized by the results that every possible control strategy can be, within an arbitrary accuracy, approximated by a fuzzy controller (for a recent survey of such results, see, e.g., [16, 26]).

The problem with this approach is that the number of rules which are necessary to represent a given control strategy with a given accuracy, grows exponentially with the increase in accuracy. As a result, for reasonable accuracy, and a reasonable number of input variable, we need astronomically many rules (see, e.g., [10, 12, 13, 15, 14]).

2 How This Problem is Solved Now

This problem has been recognized for a quite a while, and several good approaches have been proposed, including approaches by B. Kosko and L. Koczy [10, 12, 13, 15, 14]).

Kosko's main idea consists of building the rules around the *extrema* of the control function. In most practical examples, if a control function has few extrema, then we are able to approximate this control with much fewer rules than in the general case.

Koczy's idea is, crudely speaking, to reduce the number of rules by applying an appropriate transformation of the membership functions.

Both approaches work well in realistic situations when the desired control strategy is reasonably smooth (and slowly changing) outside a few critical regions where the specific strategy is important (see, e.g., [32] and references therein).

3 A Multi-Resolution Approach to Solving This Problem: Main Idea

Another possible answer to the exponential increase problem is the multi-resolution approach (see, e.g., [22, 23], and references therein). This answer is based on the following observation:

- Traditional fuzzy control technique is based on the assumption that we have the rules which directly relate the input $x = (x_1, \ldots, x_n)$ with the control, rules like "if x_1 is small, ..., and x_n is medium, then u should be large".
- In many real-life situations, however, the decision making process of expert human controllers is *hierarchical*. When given a set of measured values $x_1, \ldots, x_n$ which characterize the state of the controlled system (*plant*), the human controller does not immediately starts a control. The reason for this is that natural control rules are often formulated in terms of some parameters $y_1, \ldots, y_m$ of the system, which are difficult (or even impossible) to measure directly. Therefore, before the expert controller can come up with a reasonable control value, he has to:
  - first, to estimate the values of these parameters y_i based on the available knowledge (i.e., on the measured values x_j); and
  - only then, based on the values of y_i, produce a reasonable control u.

On both stages, the controller uses "fuzzy" rules.

The main result of this paper is that with such two-level systems, we can approximate an arbitrary control strategy by a rule base without invoking an unrealistic exponential increase in the number rules. Moreover, we will show that this is true even when on each stage, we have simple fuzzy rules, with a crisp conclusion.

Let us describe such rule bases in precise terms.

4 Towards a Formal Description of Two-Stage Rule-Based Fuzzy Control

We want to describe the expert controller's rules, both the first-stage rules, which describe the auxiliary variables $y_1, \ldots, y_m$ in terms of the inputs $x_1, \ldots, x_n$, and the second-stage rules, which describe the control u in terms of y_j.

4.1 Description of Each Stage: Traditional Approach

Since we decided to consider only the simplest rules, for which the conclusion is crisp, the general definition of a one-stage rule base will look like this:

Definition 1. *Let N be a positive integer, and let $z_1, \ldots, z_N, t$ be variables. By a rule base which describes t in terms of z_i, we mean a finite collection $\mathcal{R}$ of rules of the type*

$$\text{"if } z_1 \text{ is } A_{r1}, z_2 \text{ is } A_{r2}, \ldots, \text{and } z_N \text{ is } A_{rN},$$

$$\text{then } t = t^{(r)}\text{"}, \quad (1)$$

where $r = 1, \ldots, R$, A_{ri} are fuzzy sets (with continuous membership functions $\mu_{ri}(z_i)$), and $t^{(1)}, \ldots, t^{(R)}$ are real numbers.

Fuzzy control technique translates these rules into a precise dependence $t = f(z_1, \dots, z_N)$ which is defined as follows:

Definition 2. *Let $\mathcal{R}$ be a rule base* (1) *which describes t is terms of z_j, and let $f_\&$ be a continuous t-norm.*

- *For each set of values $z_1, \dots, z_N$, and for each $r = 1, \dots, R$, by a degree d_r with which $r-th$ rule is fired, we mean a number $d_r = f_\&(\mu_{r1}(z_1), \dots, \mu_{rN}(z_N))$.*
- *By a function $t = f_{\mathcal{R}}(z_1, \dots, z_N)$ corresponding to the rule base $\mathcal{R}$, we mean a function*

$$t = \frac{d_1 \cdot t^{(1)} + \ldots + d_R \cdot t^{(R)}}{d_1 + \ldots + d_R}. \tag{2}$$

4.2 Description of Each Stage: New Approach Aimed at Decreasing the Number of Rules

Definitions 1 and 2 describe a traditional description of fuzzy rules, in which we have to describe the appropriate value of t for all possible combinations of z_i. The number of such combinations exponentially increases with the growth of N, so, when N increases, the necessary number of rules starts growing exponentially. To somewhat curb this growth, we can use the fact that often, the same value t is reasonable for several different combinations of z_i. For example, the fact that we use "if-then" rules naturally leads to a possibility of adding, to the rule base (1), an "else" clause

$$\text{“else } t = t^{(0)}\text{”}. \tag{3}$$

The degree d_0 to which this "else" rule is fired can be defined, e.g., as $1-(d_1+\ldots+d_R)$. This is possible only if $d_1+\ldots+d_R \le 1$; in this case, $d_1+\ldots+d_R+d_0 = 1$, and the formula (2) becomes even simpler. As a result, we arrive at the following definitions:

Definition 1′. *Let N be a positive integer, and let $z_1, \dots, z_N, t$ be variables. By a rule base which describes t in terms of z_i, we mean a finite collection $\mathcal{R}$ of R rules of type* (1) *plus an additional "else"-rule* (3).

Definition 2′. *Let $\mathcal{R}$ be a rule base* (1), (3) *which describes t is terms of z_j, and let $f_\&$ be a continuous t-norm.*

- *For each set of values $z_1, \dots, z_N$, and for each $r = 1, \dots, R$, by a degree d_r with which $r-th$ rule is fired, we mean a number $d_r = f_\&(\mu_{r1}(z_1), \dots, \mu_{rN}(z_N))$.*
- *We say that a rule base is consistent if*

$$d_1 + \ldots + d_R \le 1.$$

For a consistent rule base, the degree d_0 with which the "else" rule is fired is defined as $d_0 = 1 - (d_1 + \ldots + d_R)$.

- *By a function $t = f_{\mathcal{R}}(z_1, \dots, z_N)$ corresponding to the rule base $\mathcal{R}$, we mean a function*

$$t = d_1 \cdot t^{(1)} + \ldots + d_R \cdot t^{(R)} + d_0 \cdot t^{(0)}. \tag{4}$$

4.3 Resulting Description of a Two-Stage Control

Definition 3. *Let n and m be positive integers, let $x_1, \dots, x_n$ be input variables, let $y_1, \dots, y_m$ be auxiliary variables, and let u be the control.*

- *By a two-stage rule base which describes u in terms of x_i, we mean the collection consisting of m rules bases $\mathcal{R}_1, \dots, \mathcal{R}_m$ which describe $y_1, \dots, y_m$ in terms of x_i, and a rule base $\mathcal{R}$ which describes u in terms of $y_1, \dots, y_m$.*
- *By a function $t = f(x_1, \dots, x_n)$ corresponding to the two-stage rule base, we mean a function*

$$u = f_{\mathcal{R}}(y_1, \dots, y_m), \tag{5}$$

where, for every j from 1 to m,

$$y_j = f_{\mathcal{R}_j}(x_1, \dots, x_n). \tag{6}$$

5 Main Result

Theorem. *Let $B = [a_1, b_1] \times \ldots \times [a_n, b_n]$ be an arbitrary n-dimensional box, let $f(x_1, \dots, x_n)$ be a continuous function on this box, and let $f_\&$ be a continuous t-norm. Then, there exists a consistent two-stage rule base for which the corresponding function is exactly the given function $f(x_1, \dots, x_n)$.*

In other words, for every continuous function $f(x_1, \dots, x_n)$ on a bounded set B, there exists a system of fuzzy rules which leads, after the two-stage defuzzification, exactly to this function.

In short, every possible control strategy can be not only *approximated* by a traditional one-stage fuzzy controller, it can also be *exactly* represented by a two-stage fuzzy controller. Since the two-stage representation is exact, the required number of rules does not depend on the approximation accuracy. So, the proposed approach shows promise in *curtailing the rule base explosion.*

Comments.

- For the case of a single input variable ($n = 1$), a similar result was proven in [1, 4, 14, 20, 21, 30, 31]: by using only two rules, we can exactly represent an arbitrary continuous function $f(x_1)$ on an interval $[a_1, b_1]$.

- The *downside* of this result is that while the standard approximation results use, say, triangular membership functions that are easy to store and to process, this new scheme uses special membership functions (which, in essence, code the desired control strategy). So, while we get fewer rules, we do not automatically decrease the total amount of information that needs to be stored.

6 Proof

6.1 The Main Result on Which This Proof is Based

The proof of our theorem is based on a theorem proven by A. Kolmogorov [11] as a solution to the conjecture of D. Hilbert, formulated as the thirteenth problem [7]: one of 23 problems that Hilbert has proposed in 1900 as a challenge to the XX century mathematics.

This theorem says that an arbitrary continuous function $f(x_1, ..., x_n)$ on an n-dimensional box (of arbitrary dimension n) can be represented as a superposition of addition and functions of one variable.

The beginning of this topic can be traced to the Babylonians, who found (see, e.g., [3]) that the solutions x of quadratic equations $a \cdot x^2 + b \cdot x + c = 0$ (viewed as function of three variables a, b, and c) can be represented as superpositions of functions of one and two variables, namely, arithmetic operations and square roots. Much later, similar results were obtained for functions of five variables a, b, c, d, e, that represent the solution of quartic equations $a \cdot x^4 + b \cdot x^3 + c \cdot x^2 + d \cdot x + e = 0$. But then, E. Galois proved in 1830 that for higher order equations, we cannot have such a representation. This negative result has caused Hilbert to conjecture that not all functions of several variables can be represented by functions of two or fewer variables. Hilbert's conjecture was refuted by A. Kolmogorov and his student V. Arnold.

This result has already been used in soft computing: in 1987, it was used to produce the first proof that neural networks are universal approximators [6] (see also [18, 19]); later on, it was used to prove that an arbitrary logical operation can be approximated by a combination of standard fuzzy logical operators (see, e.g., [25, 27]).

It is worth mentioning that Kolmogorov's result is not only of theoretical value: it was used to speed up actual computations (see, e.g., [5, 24]).

6.2 The Proof Itself

In our proof, we will use a simplified (and improved) version of Kolmogorov's theorem, formulated in [28] (further improvements were later described in [29]):

Theorem. [28] *For an arbitrary box*

$$B = [a_1, b_1] \times \ldots \times [a_n, b_n],$$

every real continuous function $f : B \to R$ can be represented as

$$f(x_1, ..., x_n) = \sum_{1 \le j \le 2n+1} \chi\left(\sum_{1 \le i \le n} \psi_{ji}(x_i)\right),$$

for some continuous functions $\chi(y)$ and $\psi_{ji}(x_i)$.

In other words, every continuous function $f(x_1, \ldots, x_n)$ on a box B can be represented as

$$f(x_1, \ldots, x_n) = \chi(y_1) + \ldots + \chi(y_{2n+1}), \qquad (7)$$

where for every j from 1 to $m = 2n + 1$, we have

$$y_j = \psi_{j1}(x_1) + \ldots + \psi_{jn}(x_n). \qquad (8)$$

In view of this result, one way to prove our theorem is to show that for each of the functions (7) and (8), we can find a one-stage rule base $\mathcal{R}$ (correspondingly, $\mathcal{R}_j$), to which this very function is corresponding. Then, by combining these one-stage rules bases $\mathcal{R}$ and $\mathcal{R}_j$, we will get the desired two-stage rule base which represents exactly the original function $f(x_1, \ldots, x_n)$.

We will show how this rule base $\mathcal{R}_j$ can be constructed for the function (8); for (7), the construction is similar. Since each function $\psi_{ji}(x_i)$ is a continuous function on the interval $[a_i, b_i]$, it has a finite minimum m_{ji} and a finite maximum M_{ji}. To describe the value of this function in fuzzy terms, we construct the auxiliary membership function

$$\mu_{ji}(x_i) = \frac{1}{n} \cdot \frac{\psi_{ji}(x_i) - m_{ji}}{M_{ji} - m_{ji}}.$$

(This function is similar to an "optimization" membership function introduced by R. Bellman and L. Zadeh in [2].) Let A_{ji} indicate the fuzzy set with the membership function $\mu_{ji}(x_i)$. We want to choose the real numbers $y_j^{(1)},\ldots,y_j^{(n)}$ and $y_j^{(0)}$ in such a way that the rule base

$$\text{“if } A_{ji}(x_i), \text{then } y_j = y_j^{(i)}\text{”},\ 1\le i\le n,$$

$$\text{“else } y_j = y_j^{(0)}\text{”}$$

corresponds to the function (8). This rule base has $R=n$ regular rules and an "else" rule. For each i from 1 to n, the degree to which i-th rule is fired is equal to $d_i=\mu_{ji}(x_i)$. Since M_{ji} is a maximum of a function $\psi_{ji}(x_i)$, from our definition of $\mu_{ji}(x_i)$, it follows that $\psi_{ji}(x_i)\le M_{ji}$ and therefore, that

$$\psi_{ji}(x_i)-m_{ji}\le M_{ji}-m_{ji}$$

and $\mu_{ji}(x_i)\le 1/n$. Hence, $d_i\le 1/n$ for all i, and so $d_1+\ldots+d_n\le 1/n+\ldots+1/n=1$; therefore, this rule base is consistent.

In accordance with Definition $2'$, the function corresponding to this rule base is equal to

$$y_j=\sum_{i=1}^n y_j^{(i)}\cdot\mu_{ji}(x_i)+y^{(0)}\cdot\left(1-\sum_{i=1}^n\mu_{ji}(x_i)\right)=$$

$$\sum_{i=1}^n\left(y_j^{(i)}-y_j^{(0)}\right)\cdot\mu_{ji}(x_i)+y^{(0)}.$$

Substituting the expression for $\mu_{ji}(x_i)$ into this formula, and separating terms proportional to $\psi_{ji}(x_i)$ from the terms which do not contain ψ_{ji}, we get the following formula:

$$y_j=\sum_{i=1}^n\frac{y_j^{(i)}-y_j^{(0)}}{n\cdot(M_{ji}-m_{ji})}\cdot\psi_{ji}(x_i)+$$

$$\left[y^{(0)}-\sum_{i=1}^n\frac{(y_j^{(i)}-y_j^{(0)})\cdot m_{ji}}{n\cdot(M_{ji}-m_{ji})}\right]. \qquad (9)$$

Thus, to get the formula (8), we must choose the values $y_j^{(i)}$ so that

$$y_j^{(i)}-y_j^{(0)}=n\cdot(M_{ji}-m_{ji}),\quad 1\le i\le n, \qquad (10)$$

$$y_j^{(0)}=\sum_{i=1}^n\frac{(y_j^{(i)}-y_j^{(0)})\cdot m_{ji}}{n\cdot(M_{ji}-m_{ji})}. \qquad (11)$$

Substituting (10) into (11), we conclude that

$$y_j^{(0)}=\sum_{i=1}^n m_{ji}$$

and therefore, from (10), that

$$y_j^{(i)}=\sum_{i=1}^n m_{ji}+n\cdot(M_{ji}-m_{ji})$$

for $1\le i\le n$. For these values $y_j^{(i)}$, the function (9) indeed coincides with (8).

So, each formula (8) can be represented by a one-stage rule base, and similarly, the formula (7) can be represented by a one-stage rule base. Hence, the original function $f(x_1,\ldots,x_n)$ can be represented by a two-stage rule base. The theorem is proven.

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