A Statistical Analysis for Rule Base Reduction

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Abstract: When we take into account more input variables in a control system, the number of rules grows exponentially. To decrease the number of rules, we propose not to explicitly state the control for every combination of input variables, but to use the "otherwise" clause. In this paper, we provide a simple statistical analysis of the resulting reduction and show, on a case study, that this reduction can indeed be drastic.

Keywords: fuzzy rule base reduction

1 Introduction

When we take into acount more input variables in a control system, the number of rules grows exponentially: if we have k possible values for each of n variables, we must describe control corresponding to all k^n possible combinations of input values.

To decrease the number of rules, we propose not to explicitly state the control for every combination of input variables, but to use the "otherwise" clause.

In this paper, we provide a simple statistical analysis of the resulting reduction and show, on a case study, that this reduction can indeed be drastic.

2 Definitions and the main result

Definition. *Let V be a finite set called the set of values. The number of elements in this set will be denoted by k.*

- *By a system, we mean a function $f : V^n \to V$ for some integer n.*
- *By a traditional rule base for a system f, we mean a collection of $N = k^n$ rules*

  "if x_1 is v_1, ..., and x_n is v_n, then y is v",

  that correspond to all possible values $v = f(v_1, \ldots, v_n)$ of the function f.
- *By a reduced rule base corresponding to the value $v_0 \in V$, we mean a collection of rules of the above type which describe all the values $(v_1, \ldots, v_n)$ for which $f(v_1, \ldots, v_n) \neq v_0$, plus a rule*

  "otherwise y is v_0".

  The number of rules in a reduced rule base (not counting the otherwise rule) will be denoted by N'. The ratio $(N - N')/N$ will be called a (relative) reduction degree.

Comment. We do not count the "otherwise" rule because it is much easier to describe than all the other rules.

Theorem. *For every system, there exists a value v_0 for which the reduction degree is at least $1/k$.*

In particular, this means that when we use $k = 3$ different levels of each input variable, we can decrease the number of rules by at least 1/3. If we use $k = 5$ different levels, we can still reduce the number of rules by at least 20%, etc.

Proof. Let f be a system. To each of $N = k^n$ possible combinations of inputs, the function f assigns one of the values $v \in V$. Thus, if we denote, by $N(v)$, the number of input combinations which cor-

respond to each $v \in V$, we conclude that

$$\sum_{v \in V} N(v) = N.$$

The sum of k values $N(v)$ is equal to N, thus, at least one of these values must be $\geq N/k$. (Otherwise, their sum would be smaller than $k \cdot N/k < N$, but it is $= N$.)

Let us pick, as v_0, one of the values for which $N(v_0) \geq N/k$. Then, in the corresponding reduced rule base, the total number of rules is equal to $N' = N - N(v_0)$, and the reduction degree is equal to $(N - N')/N = N(v_0)/N$. Since $N(v_0) \geq N/k$, we conclude that the reduction degree is

$$\frac{N - N'}{N} = \frac{N(v_0)}{N} \geq \frac{N/k}{N} = \frac{1}{k}.$$

Q.E.D.

3 Case study: nondestructive testing of aerispace structures

One of the most important characteristics of the plane is its weight: every pound shaved off the plane means a pound added to the carrying ability of this plane. As a result, planes are made as light as possible, with their "skin" as thin as possible. However, the thinner the layer, the more vulnerable is the resulting structure to stresses and faults, and flight is a very stressful experience. Therefore, even minor faults in the plane's structure, if undetected, can be disastrous. To avoid possible catastrophic consequences, before the flight, we must thoroughly check the structural integrity of the plane.

Some faults, like cracks, holes, etc., are external, and can, therefore, be detected during the visual inspection. However, to detect internal faults (cracks, holes, etc.), we must somehow scan the inside of the thin plate that forms the skin of the plane. This skin is not transparent to light or to other electromagnetic radiation; very energetic radiation, e.g., X-rays or gamma-rays, can go through the metal, but it is difficult to use on such a huge object as a modern plane.

The one thing that easily penetrates the skin is vibration. Therefore, we can use sound, ultrasound, etc., to detect the faults. Usually, a wave easily glosses over obstacles whose size is smaller than its wavelength. Therefore, since we want to detect the smallest possible faults, we must choose the sound waves with the smallest possible wavelength, i.e., the largest possible frequency. This frequency is usually higher than the frequencies that we hear, so it corresponds to ultrasound.

Ultrasonic scans are indeed one of the main nondestructive NDE tools; see, e.g, [2, 3].

In *nondestructive testing* of structural integrity, we send an ultrasonic signal to the tested system, and measure the resulting vibration at different points. Our goal is to detect the points where the cracks or other possible faults are.

4 Modal approach to nondestructive testing: idea

In nondestructive testing of aerospace structures, we would like to process the measurement information as fast as possible. For large structures, however, with lots of sensors, and with a highly dynamical (ultrasonic) signal, we get a large amount of data, and processing this data as a whole would take too long.

To decrease the data processing time, we can use the known fact that a vibration of a mechanical structure can be represented as a combination of different independent *modes* (corresponding to different eigenvalues of the corresponding matrix). Therefore, after measuring the vibrations, it is reasonable to separate the measurement results into results corresponding to different modes, and process each mode independently.

For each vibration mode, we can estimate the energy density at each point; if this measured energy density is higher than in the original (undisturbed) state, this is a good indication that a fault may be located at this point. The larger the increase in energy density, the larger the probability of a fault. As a result, for each point x, and for each mode i, we get the probability $p_i(x)$ that, based on the measurements related to this mode, there is a fault at a point x.

We need to combine these probabilities into a probability $p(x)$ that there is a fault at x.

5 Main problem of modal approach

The modal approach, as described above, requires the use of *probabilities*:

First, we need to describe how the probability of the fault at a certain point depends on the excess energy at this point.

Second, we must transform the probabilities coming from different modes into a single probability value.

Our experience shows that using wrong probabilities can lead to errors of both possible type:

- time-consuming *false positives*, when a fault is claimed in a location where there is no fault at all, and
- dangerous *false negatives*, when the existing fault is not detected at all.

It is therefore very important to get these probabilities right. How can we get these probabilities?

In some cases, we have enough statistics, so we can determine these probabilities from the analysis of the experimental data. However, often, we do not have that statistics: e.g., when we start a *new method*, more accurate measurements, etc., there is not yet enough statistics to determine the probabilities. Similarly, when we apply the existing method to a *new object* (e.g., to a Space Station), there is not yet enough statistics.

6 Expert knowledge can supplement the missing statistics

Since we cannot determine the probabilities solely from experiment, we must therefore use some additional expert knowledge to supplement our experimental data.

We have successfully done that, by using different soft computing techniques such as fuzzy techniques, neural networks, and genetic algorithms (see, e.g., [6, 7, 9]), and we got pretty good results.

7 Experimental results

As a case study, we applied the modal approach to the problem of non-destructive evaluation of structural integrity of Space Shuttle's vertical stabilizer.

To test the applicability of our method, we applied this techniques to measurement results for pieces with known fault locations.

The methods that we came up with detected all the faults in $>70\%$ of the cases, much larger proportion than with any previously known techniques (for details, see [1, 7, 10]).

8 Remaining problems

There are two main problems with this result:

- first, due to the fact that we used several different (and reasonably complicated) formalisms, the resulting computational models are rather time-consuming and not very intuitive;
- second, although we got better fault detection that all previously known methods, but there is a still quite some room for improvement.

9 New idea: the use of fuzzy rule base

The main problem we face is the problem of complexity of the computational models we use. Complex models are justified in such areas as fundamental physics, when simpler first approximation models have been tried and turned out not exactly adequate. However, in our case, the computational models are chosen not because simpler models have been tried, but because these complex models were the only ones which we could find which fit our data and are consistent with the expert knowledge.

The very fact that a large part of our knowledge comes from expert estimates, which have a high level of uncertainty, makes us believe that within this uncertainty, we can find simpler computational models which will work equally well. How can we find such models?

A similar situation, when unnecessarily complex models were produced by the existing techniques, started the field of fuzzy logic. Namely, L. Zadeh proposed to use, instead of traditional analytical models, new simplified models based on the direct formalization of expert's knowledge.

In view of the success of fuzzy techniques, it is reasonable to use a similar approach in fault detection as well. Let us first describe the corresponding rules.

10 Expert rules for fault detection

For each location, as a result of the measurements, we get five different values of the excess energy $E_1, \ldots, E_5$ which correspond to 5 different modes. An expert can look at these values and tell whether we have a definite fault here, or a fault with a certain degree of certainty, or definitely no fault at all.

Before we formulate the expert rules, we should note that for each node, the *absolute* values of excess energy are not that characteristic because, e.g., a slight

increase or decrease in the original activation can increase or decrease all the values of the excess energy, while the fault locations remain the same. Therefore, it is more reasonable to look at *relative* values of the excess energy. Namely, for each mode i, we compute the mean square average σ_i of all the values, and then divide all values of the excess energy by this means square value to get the corresponding relative value of the excess energy $x_i = E_i/\sigma_i$.

In accordance with the standard fuzzy logic methodology, we would like to describe some of these values as "small positive" (SP), some as "large positive" (LP), etc. To formalize these notions, we must describe the corresponding membership functions $\mu_{SP}(x)$ and $\mu_{LP}(x)$.

Some intuition about the values x_i comes from the simplified situation in which the values of excess energy E_i are random, following a normal distribution with 0 average. In this simplified situation, the mean square value σ_i is (practically) equal to the standard deviation of this distribution. For normal distributions, deviations which exceed $2\sigma_i$ are rare and are therefore usually considered to be definitely large; on the hand, deviations which are smaller than the average σ_i are, naturally, definitely small. Deviations $E_i \geq 2\sigma_i$ correspond to the values $x_i = E_i/\sigma_i \geq 2$, and deviations $E_i \leq \sigma_i$ correspond to $x_i = E_i/\sigma_i \leq 1$. Therefore, can conclude that values $x_i \geq 2$ are definitely large, and positive values $x_i \leq 1$ are definitely small.

So, for the fuzzy notion "small", we know that:

- values from 0 to 1 are definitely small, i.e., $\mu_{SP}(x_i) = 1$ for these values, and
- values 2 and larger are definitely not small, i.e., $\mu_{SP}(x_i) = 0$ for these values.

These formulas determine the value of the membership function for all positive values of x_i, except for the values from 1 to 2. In accordance with the standard fuzzy techniques, we use the simplest – linear – interpolation to define $\mu_{SP}(x_i)$ for values from this interval, i.e., we take $\mu_{SP}(x_i) = 2 - x_i$ for $x_i \in [1, 2]$.

Similarly, we define the membership function for "large" as follows: $\mu_{LP}(x_i) = 0$ for $x_i \in [0, 1]$; $\mu_{LP}(x_i) = x_i - 1$ for $x_i \in [1, 2]$; and $\mu_{LP}(x_i) = 1$ for $x_i \geq 2$.

Similarly, we describe the membership functions corresponding to "small negative" (SN) and "large negative" (LN): in precise terms, for $x_i < 0$, we set $\mu_{SN}(x_i) = \mu_{SP}(|x_i|)$ and $\mu_{LN}(x_i) = \mu_{LP}(|x_i|)$.

This takes care of fuzzy terms used in the condition of expert rules. To describe the conclusion, we determined that experts use 5 different levels of certainty, from level 1 to level 5 (absolute certainty). We can identify these levels with numbers from 0.2 to 1.

Now, we are ready to describe the rules.

1. *If the "total" excess energy $x_1 + \ldots + x_5$ attains its largest possible value, or is close to the largest possible value (by ≤ 0.06), then we definitely have a fault at this location (this conclusion corresponds to level 5).*

2. *If all 5 modes show increase, then we have a level 4 certainty that there is a fault at this location.*

3. *If 4 modes show increase, and one mode shows small or large decrease, then level 4.*

4. *If 3 modes show increase and 2 show small decreases then level 4.*

5. *If 3 modes show increase, and we have either 1 small and 1 large decrease, or 2 large decreases, then level 3.*

6. *If 2 modes show large increase and 3 modes show small decrease, then level 3.*

7. *If 2 modes show large increase, 1 or 2 modes show large decrease, and the rest show decrease, then level 2.*

8. *If 1 mode shows large increase, 1 mode shows small increase, and 3 modes show small decrease, then level 2.*

9. *In all other cases, level 1.*

11 The problem with this rule base and how we solve it

The technique of fuzzy modeling and fuzzy control enables us to transform rule bases (like the one above) into an algorithm which transforms the inputs $x_1, \ldots, x_n$ into a (defuzzified) value of the output y. In principle, we can apply this technique to our rule base, but the problem is that we will need too many rules. Indeed, standard rules are based on the conditions like "if x_1 is A_1, ..., and x_n is A_n, then y is B". In our case, we have 5 input variables, each of which can take 4 different fuzzy values (LN, SN,

SP, and LP). So, to describe all possible combinations of inputs, we must use $4^5 = 1,024$ rules. It is doable, but it is definitely not the simplification for which we were looking.

To decrease the number of the resulting rules, we can use the fact that all the rules do not distinguish between different modes. Therefore, if we permute the values x_i (e.g., swap the values x_1 and x_2), the expert's conclusion will not change. Hence, instead of considering all possible combinations of x_i, we can first apply some permutation to decrease the number of possible combinations. One such permutation is *sorting* the values of x_i, i.e., re-ordering these values in the decreasing order. Let us show that if we apply the rules to thus re-ordered values, then we can indeed drastically decrease the number of resulting fuzzy rules.

Let $y_1 \geq y_2 \ldots \geq y_5$ denote the values $x_1, \ldots, x_5$ re-ordered in decreasing order. Let us show how, e.g., Rules 2, 3, and 4 from the above rule base can be reformulated in terms of these new values y_i:

Rule 2. To say that all five values x_i are positive is the same as to say that the smallest of these values is positive, so the condition of Rule 2 can be reformulated as $y_5 > 0$.

Rule 3. When 4 modes are positive and the fifth is negative, it means that $y_4 > 0$ and $y_5 < 0$.

We can notice that since Rules 2 and 3 have the same conclusion, they can be combined into a single rule with a new (even simpler) condition $y_4 > 0$. (Indeed, we either have $y_5 > 0$ and $y_5 \leq 0$; if $y_4 > 0$ and $y_5 > 0$, then the conclusion is true because of Rule 2; if $y_4 > 0$ and $y_5 < 0$, then the conclusion is true because of Rule 3.)

Rule 4. Similarly, its condition can be reformulated as $y_3 > 0$, y_4 is SN, and y_5 is SN.

As a result, we get the following new (simplified) rule base:

1. *If the "total" excess energy $y_1 + \ldots + y_5$ attains its largest possible value, or is close to the largest possible value (by ≤ 0.06), then level* 5.

2. *If $y_4 > 0$, then level* 4.

3. *If $y_3 > 0$, y_4 is SN, and y_5 is SN, then level* 4.

4. *If $y_3 > 0$, $y_4 < 0$, and y_5 is LN, then level* 3.

5. *If y_2 is LP, $y_3 < 0$, and y_5 is SN, then level* 3.

6. *If y_2 is LP, $y_3 < 0$, and y_5 is LN, then level* 2.

7. *If y_1 is LP, y_2 is SP, $y_3 < 0$, y_4 is SN, and y_5 is LN, then level* 2.

8. *In all other cases, level* 1.

To transform these fuzzy rules into a precise algorithm, we must select a fuzzy "and"-operation (t-norm) and a fuzzy "or"-operation (t-conorm), e.g., $\min(a, b)$ and $\max(a, b)$, and a defuzzification; in our paper, we use centroid defuzzification.

For each rule (except for the last one), we can compute the degree of satisfaction for each of the conditions. The rule is applicable if its first condition holds, *and* the second condition holds, etc. So, to find the degree with which the rule is applicable, we apply the chosen "and"-operation to the degrees with which different conditions of this rule hold.

For each level > 1, we have two rules leading to this level. The corresponding degree of certainty is achieved if either the first *or* the second of these rules is applicable. Therefore, to find a degree to which this level is justified, we must apply the chosen "or"-operation to the degrees to which these two rules are applicable.

As a result, we get the degrees $d(l)$ with which we can justify levels $l = 2 \div 5$. Since the last rule (about level 1) says that this rule is applicable when no other rule applies, we can compute $d(1)$ as $1 - d(2) - \ldots - d(5)$. Now, centroid defuzzification leads to the resulting certainty $1 \cdot d(1) + 2 \cdot d(2) + \ldots + 5 \cdot d(5)$. This is the value that the system outputs as the degree of certainty (on a 1 to 5 scale) that there is a fault at a given location.

12 Experimental results

We have applied the resulting fuzzy model to the beams with known fault locations. The results are as follows:

When there is *only one fault*, this fault can be determined as the location where the degree of certainty attains its largest value 5. This criterion leads to a *perfect fault localization*, with no false positives and no false negatives.

When there are *several faults*, all the faults correspond to locations with degree 4 or larger. This criterion is not perfect; it *avoids the most dangerous errors of false negatives* (i.e., all the faults are

detected), but it has false positives, i.e., sometimes faults are wrongly indicated in the areas where there are none.

To make the fuzzy algorithm better, we take into consideration that the vibration corresponding to each mode has points in which the amplitude of this vibration is 0. The corresponding locations are not affected by this mode and therefore, the corresponding excess energy values cannot tell anything about the presence or absence of a fault. Therefore, it makes sense to only consider those values x_i for which the corresponding mode energy is at least, say, 10% of its maximum. If we thus restrict the values x_i, then the number of false positives decreases.

We tried different t-norms and t-conorms. So far, we have not found a statistically significant difference between the results obtained by using different t-norms and t-conorms; we hope that for more complicated examples of 2D surfaces with faults, we will be able to detect this difference, and thus, find t-norms and t-conorms which are the best for fault detection.

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