USING FFT-BASED DATA PROCESSING TECHNIQUES TO CHARACTERIZE ASPHALTIC CONCRETE MIXTURES

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ABSTRACT

A natural way to test the quality of a pavement is to send signals with different frequencies through the pavement and compare the results with the signals passing through an ideal pavement. For this comparison, we must determine how, for the corresponding mixture, the elasticity E depends on the frequency f in the range from 0.1 to 10^5 Hz. It is very expensive to perform measurements in high frequency area (above 20 Hz). To avoid these measurements, we can use the fact that for most of these mixtures, when we change a temperature, the new dependence changes simply by scaling. Thus, instead of performing expensive measurements for different frequencies, we can measure the dependence of E on moderate frequencies f for different temperatures, and then combine the resulting curves into a single "master" curve. In this paper, we show how DSP techniques can help to automate this "combination".

1. PRACTICAL PROBLEM

Many materials are *viscoelastic*, i.e., they possess both the elastic property of a solid and the viscous behavior of the liquid. Examples of such materials range from asphaltic concrete mixtures used in road pavement to elastomers used in aerospace industry.

In many applications of these materials, it is desirable to use non-destructive techniques to test the structural integrity and the properties of the corresponding structures. In these techniques, we send signals to the structure, we measure the signals that passed through the structure, and we analyze this detected signal to see if the structure has any faults. For this analysis, we need to know how, for the corresponding material, the elasticity E depends on the frequency f. To detect different types of faults, we must know this dependence for the frequency f in the range from 0.1 to 10^5 Hz. It is very expensive to perform measurements in high frequency area above 20 Hz.

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To avoid these measurements, we can use the fact that many practically used viscoelastic materials – including most asphaltic concrete mixes and many elastomers used in the aerospace industry – are *thermally simple*, meaning that when we change a temperature, the new dependence changes simply by scaling, i.e.,

$$E'(f) = E(\lambda \cdot f). \tag{1}$$

Thus, instead of performing expensive measurements for different frequencies, we can measure the dependence of E on moderate frequencies f for different temperatures, and then try to apply appropriate scaling to combine the resulting curves into a single "master" curve.

To describe frequencies ranging from 0.1 to 10^5 Hz, researchers usually use logarithmic scale, i.e., they use the logarithm $F = \ln(f)$ instead of f. In the logarithmic scale, scaling is described as a shift, so the equation (1) takes an even simpler form:

$$E'(F) = E(F + c), \tag{2}$$

where $c = \ln(\lambda)$ is the corresponding shift. In the logarithmic scale, the above idea can be reformulated as follows: we measure the dependence of E on F for different temperatures, and then shift the resulting curves in horizontal direction together so that they form a single curve; see, e.g., [1, 4, 5, 6, 10].

This approach has been successfully used; the problem is that at present, the shifts are done manually by an expert. It is desirable to automate this process.

The problem of determining a shift c between two known functions $E(F)$ and $E'(F)$ occurs not only in the analysis of asphaltic concrete mixtures; a similar problem of time delay estimation is well known in digital signal processing. In this paper, we will show how the corresponding DSP methods can be used to combine the curves corresponding to different temperatures.

2. CLASSICAL TIME-DELAY ESTIMATION TECHNIQUES: DESCRIPTION

Given measurements $E(F)$ and $E_1(F)$ and the fact that $E'(F) \approx E(F+c)$, how do we determine the unknown shift c? The traditional DSP approach to solving the similar time-delay estimation problem is to estimate c as the shift after which the correlation between $E(F+c)$ and $E'(F)$ is the largest, i.e., equivalently, for which the cross-correlation function $R(c) \stackrel{\text{def}}{=} \int E'(F) \cdot E(F+c)\, dF$ attains its largest possible value.

We usually have the measurement results $E(F)$ and $E'(F)$ at equally spaced values

$$F_1, F_2 = F_1 + \Delta F, F_3 = F_2 + \Delta F, \dots,$$

i.e., we know the values $E_i \stackrel{\text{def}}{=} E(F_i)$ and $E'_i \stackrel{\text{def}}{=} E'(F_i)$. In this case, the above integral can be approximated by the corresponding discrete sum. So, $c = k \cdot \Delta F$, where k is the value at which $r(k) \stackrel{\text{def}}{=} \sum\limits_i E'_i \cdot E_{i+k}$ attains its largest value.

This is the method used, e.g., in Very Large Baseline Interferometry (VLBI) to determine the time shift between the signals from the same radiosource received by two radiotelescopes; see, e.g., [3, 8].

To use this method, for n measurement results, we need to use n computational steps to compute each of n values $r(k)$; thus, overall, we need $n \cdot n = O(n^2)$ computational steps. For large n, this may take too long. To speed up the signal processing, we can implement the cross-correlation method in the frequency domain. Specifically, we can use the known formula for the Fourier transform of the cross-correlation function and thus, compute $R(c)$ as follows:

- first, we use the FFT algorithm to $E(F)$ and $E'(F)$, resulting in $A(\omega)$ and $A'(\omega)$;
- second, for each ω, we compute the product

$$P(\omega) \stackrel{\text{def}}{=} A'(\omega) \cdot A^*(\omega);$$

- finally, we apply the inverse FFT to $P(\omega)$.

Since computing FFT requires $O(n \cdot \log(n))$ computational steps [2] (which is much fewer than $O(n^2)$), for large n, this frequency-domain technique is much faster.

3. CLASSICAL TIME-DELAY ESTIMATION TECHNIQUES: FIRST TEST

We started our research by applying the above-described classical techniques to the pavement testing data. Specifically, we tested cylinders of 2 in radius and 2 in height on 5 different temperatures: 104 F, 75 F, 60 F, 40 F, and 14 F. All samples were tested on 10 different frequencies: 20, 10, 5, 2, 1, 0.5, 0.2, 0.1, 0.05, and 0.02 KHz.

Since there is little intersection between non-neighboring curves, we only matched each data set with the data set corresponding to the two neighboring temperatures. In general, as a reference $E(F)$, we took the room-temperature (75 F) measurements.

To gauge the quality of the resulting shift estimates, we used estimates provided by the experts.

At first, we got a useless result: for all compared pairs, the maximum value of $r(k)$ is attained at $k = 0$.

4. PRE-PROCESSING: MOTIVATIONS, ALGORITHM, AND RESULTS

By looking at the data, we could see our first test failed. Indeed, the classical technique uses the values $E(F)$ for all F. However, e.g., the values E_i that are larger than all the values E'_j will never be matched in any shift, so we can as well replace them by 0s. Similarly, if a value E_i is smaller than all the values E'_j, it will never match with any value E'_j, so we can also replace it by 0.

We therefore applied the following pre-processing to our data:

- first, for each of two data sequences E_i and E'_i, we compute the largest and the smallest values

$$M \stackrel{\text{def}}{=} \max E_i, \;\; m \stackrel{\text{def}}{=} \min E_i,$$

$$M' \stackrel{\text{def}}{=} \max E'_i, \;\; m' \stackrel{\text{def}}{=} \min E'_i;$$

- then, we compute the thresholds $M_0 \stackrel{\text{def}}{=} \min(M, M')$ and $m_0 \stackrel{\text{def}}{=} \max(m, m')$;
- finally, we replace all the values E_i and E'_i that are above M_0 or below m_0 with zeros.

After this pre-processing, we applied the classical time-delay estimation techniques, and got reasonable shift values.

5. REMAINING PROBLEMS

In spite of the success of the classical time-delay estimation techniques enhanced by the above-described pre-processing, there are two problems with this approach:

- first, the resulting values were too noise-prone: the cross-correlation function $r(k)$ did attain maximum at the right value k, but the values of $r(k')$ for some $k' \neq k$ was $\approx 90\%$ of this maximum; so with less accurate measurements, we will not be able to correctly estimate the shift; and real-life data is usually much noisier that our results from a university lab;

- second, we only determine the shift with the accuracy of ΔF, which is much less accurate than experts can determine based on the same data.

In this paper, we show how to resolve both problems.

6. OPTIMAL TIME-DELAY ESTIMATION IN THE PRESENCE OF NOISE

Let us first make the method less sensitive to noise.

In the ideal no-noise situation, when $E'(F) = E(F+c)$, then the Fourier transforms of these two signals are related by the formula $A'(\omega) = R(\omega) \cdot A(\omega)$, where

$$R(\omega) \stackrel{\text{def}}{=} \exp(2\pi \cdot \mathrm{j} \cdot \omega \cdot c).$$

Thus, in the no-noise case, to determine the shift, we can compute the ratio $A'(\omega)/A(\omega)$ of the signals' FFTs (which is equal to $R(\omega)$) and apply inverse FFT to this ratio. As a result, we get a delta-function $\delta(F - c)$, whose only non-zero value is at $F = c$.

Due to noise, we only have an approximate relation between the signals' Fourier transforms:

$$A'(\omega) \approx R(\omega) \cdot A(\omega).$$

By definition of $R(\omega)$, we also know that $|R(\omega)|^2 = 1$. Thus, for each ω, to determine the best estimate for $R(\omega)$, we can use the Least Squares Method and find the value R for which $|A'(\omega) - R(\omega) \cdot A(\omega)|^2 \to \min$ under the condition that $|R(\omega)|^2 = 1$.

Applying Lagrange multiplier technique and differentiating the resulting objective function, we arrive at an explicit formula for the optimal estimate:

$$R_{\text{opt}}(\omega) = \frac{A'(\omega) \cdot A^*(\omega)}{|A'(\omega)| \cdot |A^*(\omega)|}. \tag{3}$$

In the ideal non-noise case, the inverse Fourier transform $I(F)$ of this ratio is equal to the delta-function $\delta(F - c)$, i.e., $I(F) = 0$ everywhere except for the point $F = c$. In the presence of noise, we expect the values of $I(F)$ to be slightly different from the delta-function, but still, the value $|I(c)|$ should be much larger than all the other values of this function $I(F)$. So, we arrive at the following *algorithm* for determining the shift c:

- first, we apply FFT to the original signals $E(F)$ and $E'(F)$ and compute their Fourier transforms $A(\omega)$ and $A'(\omega)$;
- on the second step, we compute the ratio (3);
- on the third step, we apply the inverse FFT to the ratio $R_{\text{opt}}(\omega)$ and compute its inverse Fourier transform $I(F)$;
- finally, on the fourth step, we determine the desired shift c as the point for which $|I(F)|$ takes the largest possible value.

The only difference between this algorithm and the classical cross-correlation technique is that instead of the product of FFTs, we consider the "normalized" product (3). This technique works well in image referencing [7, 9].

For the pavement data, we got the same optimum location as the classical method, but with less error sensitivity: all the non-optimal values $I(F_{k'})$ with $k' \neq k$ do not exceed the 25% of the maximum. So, even if the noise level increases (up to 4 times), we will still be able to find the correct maximum.

7. TOWARDS BETTER ACCURACY IN TIME-DELAY ESTIMATION: SUB-PIXEL ACCURACY

Once we found out that the desired shift is approximately equal to $k \cdot \Delta F$, i.e., that $A'_i \approx A_{i+k}$, how can we then determine the shift with a better accuracy? In other words, we would like to find $\alpha \in [-1, 1]$ such that A'_i is as close as possible to $A_{i+k+\alpha}$. To estimate $A_{i+k+\alpha}$, we can use linear extrapolation

$$A_{i+k+\alpha} \approx A_{i+k} + \alpha \cdot (A_{i+k+1} - A_{i+k}).$$

The describe the best match, it is reasonable to consider the Least Squares Method, i.e., to minimize $\sum\limits_i (A'_i - A_{i+k+\alpha})^2$:

$$\sum_i (A'_i - A_{i+k} - \alpha \cdot (A_{i+k+1} - A_{i+k}))^2 \to \min.$$

Differentiating by α and equating the derivative to 0, we can an explicit formula for the optimal value α:

$$\alpha = \frac{\sum\limits_i (A'_i - A_{i+k}) \cdot (A_{i+k+1} - A_{i+k})}{\sum\limits_i (A_{i+k+1} - A_{i+k})^2}. \tag{5}$$

The actual shift is then estimated as $(k + \alpha) \cdot \Delta F$.

This methods does indeed lead to a good match for the pavement data.

8. REMAINING MINOR PROBLEM

Combining the above techniques for decreasing noise and for determining shift with better accuracy, we get a reasonably good algorithm for shift detection.

Remaining problem: it still takes some time. Can we make it even faster?

In the above algorithms, the main computation time is taken by FFT. So, the above problem can be reformulated as follows: can we get good quality estimates for the shift c without using FFT?

9. CAN WE AVOID FFT AND STILL GET A REASONABLE ESTIMATE: FIRST IDEA

As we mentioned, usually, the dependence of E and E' on F is monotonic. Thus, for every value $\widetilde{E}$ from the interval $[m_0, M_0]$, there exists at most one value $F(\widetilde{E})$ for which $E(F) = \widetilde{E}$. Similarly, there exists at most one value $F'(\widetilde{E})$ for which $E'(F') = \widetilde{E}$.

In the idealized no-noise case, for every F, the value $E'(F)$ is exactly equal to $E(F + c)$. In particular, for $F = F'(\widetilde{E})$, we have

$$\widetilde{E} = E'(F'(\widetilde{E})) = E(F'(\widetilde{E}) + c).$$

Since $F(\widetilde{E})$ is the only value for which $E(F) = \widetilde{E}$, we thus conclude that $F(\widetilde{E}) = F'(\widetilde{E}) + c$. So, in this idealized case, we can determine c as $F(\widetilde{E}) - F'(\widetilde{E})$.

In reality, the values $E'(F)$ and $E(F + c)$ are only approximately equal. Thus, the shift c is only approximately equal to the difference $F(\widetilde{E}) - F'(\widetilde{E})$. We can repeat the same argument for the values $F(\bar{E})$ and $F'(\bar{E})$ corresponding to different values $\bar{E}$, and get difference estimates $F(\bar{E}) - F'(\bar{E})$ for the desired shift c.

Let $E^{(1)}, \ldots, E^{(K)}$ be the values for which we perform these computations. We want to find a single estimate c for the shift which is close to all the resulting K estimates $F(E^{(i)}) - F'(E^{(i)})$. It is reasonable to use the Least Squares Method to determine this value c. So, we must find c for which

$$(c - (F(E^{(1)}) - F'(E^{(1)})))^2 + \ldots +$$
$$(c - (F(E^{(K)}) - F'(E^{(K)})))^2 \to \min_c .$$

Differentiating by c and equating the derivative to 0, we conclude that

$$c = \frac{(F(E^{(1)}) - F'(E^{(1)})) + \ldots + (F(E^{(K)}) - F'(E^{(K)}))}{K},$$

i.e., that the shift can be estimate as the average horizontal shift between the two curves. Thus, we arrive at the following algorithm for finding the shift c between the two curves $E(F)$ and $E'(F)$:

- we pick several values $E^{(1)}, \ldots, E^{(K)}$ – e.g., the actual values of E_i and E'_i within the interval $[m_0, M_0]$;
- for each of these values $E^{(i)}$, we find the values $F(E^{(i)})$ and $F'(E^{(i)})$ for which $E(F(E^{(i)})) = E'(F'(E^{(i)})) = E^{(i)}$, and compute the difference $c_i = F(E^{(i)}) - F'(E^{(i)})$; specifically:
  - due to monotonicity, we can use bisection [2] find the values F_j and F'_j for which $E(F_j) \approx E^{(i)}$ and $E'(F'_j) \approx E^{(i)}$, and then
  - use linear interpolation to get the more accurate values of $F(E^{(i)})$ and $F'(E^{(i)})$;
- finally, as an estimate c for the shift, we take the arithmetic average of K values $c_1, \ldots, c_K$.

How many computational steps do we need for this method? We must repeat the computations for $K \approx 2n$ different values E_i. For each i, bisection requires $O(\log(n))$ computational steps. Thus, overall, we need $2n \cdot O(\log(n)) = O(n \cdot \log(n))$ computational steps.

Asymptotically, it is the same amount as FFT, but in practice, this method is faster, because for the new method the multiplicative constant in $O(n \cdot \log(n))$ is smaller.

This methods indeed leads to a reasonable combination of curves corresponding to different temperatures. How does it compare with the FFT techniques? Our experiments show that for pavement data, both methods are of approximately the same quality:

- in some examples, the resulting Means Square Error between $E'(F)$ and $E(F + c)$ is smaller for the FFT-based techniques;
- in other examples, it is smaller for the average.

10. CAN WE AVOID FFT AND STILL GET A REASONABLE ESTIMATE: SECOND IDEA

Another idea comes from the observation that in all our cases, the first (integer) approximation k to the desired shift is equal to the number of top zeros after the above-described pre-processing. This observation makes sense, because the data is almost monotonic.

So, we arrive at the following new algorithm:

- first, we perform pre-processing as described in Section 4;
- then, we determine k as the number of top zeros;
- finally, we adjust the integer value k by using the formula (5).

This method requires only $O(n)$ steps, which is much smaller than $O(n \cdot \log(n))$. As we have mentioned, in all our examples, it led to exactly the same results as the FFT-based techniques.

11. CONCLUSIONS

In this paper, we suggest to use temperature changes to determine viscoelastic material properties. In the logarithmic frequency scale, the temperature sensitivity becomes a linear shift. An FFT-based procedure is used to find this shift.

This procedure is similar to the cross-correlation method implemented in the frequency domain.

We also describe how we can further speed up computations by avoiding FFT.

12. ACKNOWLEDGMENTS

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14. APPENDIX: DERIVATION OF THE FORMULA (3)

For conditional minimization, according to the Lagrange multiplier technique, the minimum of a function $f(x)$ under the condition $g(x) = 0$ is attained when for some real number λ, the auxiliary function $f(x) + \lambda \cdot g(x)$ attains its unconditional minimum; this value λ is called a *Lagrange multiplier.*

For our problem, the Lagrange multiplier technique leads to the following unconditional minimization problem:

$$A'(\omega) \cdot A'^*(\omega) - R^* \cdot A'(\omega) \cdot A^*(\omega) -$$
$$R \cdot A'^*(\omega) \cdot A(\omega) + R \cdot R^* \cdot A(\omega) \cdot A^*(\omega) +$$
$$\lambda \cdot (R \cdot R^* - 1) \to \min . \tag{6}$$

We want to find the value of the complex variable R for which this expression takes the smallest possible value. A complex variable is, in effect, a pair of two real variables, so the minimum can be found as a point at which the partial derivatives with respect to each of these variables are both equal to 0. Alternatively, we can represent this equality by computing the partial derivative of the expression (6) relative to R and R^*. If we differentiate (6) relative to R^*, we get the following linear equation:

$$-A'(\omega) \cdot A^*(\omega) + R \cdot A(\omega) \cdot A^*(\omega) +$$
$$\lambda \cdot R = 0. \tag{7}$$

From this equation, we conclude that

$$R = \frac{A'(\omega) \cdot A^*(\omega)}{A(\omega) \cdot A^*(\omega) + \lambda}. \tag{8}$$

The coefficient λ can be now determined from the condition that the resulting value R should satisfy the equation $|R|^2 = 1$. The denominator $A(\omega) \cdot A^*(\omega) + \lambda$ of the equation (8) is a real number, so instead of finding λ, it is sufficient to find a value of this denominator for which $|R|^2 = 1$. One can easily see that to achieve this goal, we should take, as this denominator, the absolute value of the numerator, i.e., the value

$$|A'(\omega) \cdot A^*(\omega)| = |A'(\omega)| \cdot |A^*(\omega)|. \tag{9}$$

For this choice of a denominator, the formula (7) takes the form (3).