Fuzzy Prediction Models in Measurement

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***Abstract*-Measurement is a fundamental method of scientific cognition, which employs a variety of different models. The paper investigates a feasibility of fuzzy models application in measurement procedures. It considers the problem of measurement information fusion from different sources, when one of the sources provides predictions regarding approximate values of the measured variables or their combinations. Typically this information is given by an expert but may be mined from available data also. This information is formalized as fuzzy prediction models and is used in combination with the observation results to improve the measurement accuracy. The properties of the modified estimates are studied in comparison with the conventional ones. The conditions when fuzzy models application can achieve a significant accuracy gain are derived, the gain value is evaluated, the recommendations on fuzzy prediction model production and formalization in practical applications are given.**

Keywords: fuzzy prediction, measurement science, accuracy gain

I. Introduction

Measurement is the most fundamental method of science in obtaining knowledge and in controlling systems. Despite numerous publications, we are just beginning to understand the philosophical processes occurring at the measurement and to express these in terms of a formalized mathematical model [6]. This model derivation occurs as a dynamic multi-step transformation usually conducted in a loop. As Bertrand Russell pointed out in [27] "In arriving at a scientific law there are three main stages: The first consists in observing the significant facts; the second in arriving at a hypothesis, which, if it is true, would account for these facts; the third is deducing from this hypothesis consequences which can be tested by observation. If the consequences are verified, the hypothesis is provisionally accepted as true, although it will usually require modification later on as a result of the discovery of further facts". In recent decades the hypothesis formed has been more commonly referred to as a model, a conceptual framework whose validity could be updated or modified as further information is gathered. From an algorithmic prospective, the measurement process could be considered as updating models of the systems under measurement and estimating their parameters (see fig.1). P. Sydenham [30] gives the following list of the major model types commonly applied in measurement:

- linguistic, in which the relationships are expressed in words and sentences,
- iconic, in which the relationships are expressed in terms of graphs, charts, etc,
- mathematical, in which relationships between aspects of the system to be measured are expressed in terms of equations.

Each represents only aspects of the system under measurement and their information contents vary widely. All of these types have their advantages and drawbacks, which are discussed further in section III. Each of them has a role to play in representing the real system under measurement. However, linguistic models in measurement practice are commonly considered as less precise and somewhat mathematically immature. This approach does not coincide with the models applied by L.Zadeh in his theory of generalized definability [39] (see fig.1). Zadeh introduces a hierarchy, which involves five different levels of definibility:

(1) c(risp) definability, e.g. a number, a linear system, or a Gaussian distribution, which represents the lowest level of classification,

(2) f(uzzy) definability, a concept associated with a fuzzy set and a membership function, e.g. a small number, similarity,

(3) f.g(ranular) definability, where the values of attributes are granulated, e.g. a concept of statistical independence,

(4) P(recisiated) N(atural) L(anguage) definability, where propositions drawn from a natural language are applied, e.g. the value of temperature in point A usually approximately follows up changes in an amount of carbon dioxide in the released.,

(5) G(eneralized) C(onstraint) Language, which is maximally expressive.

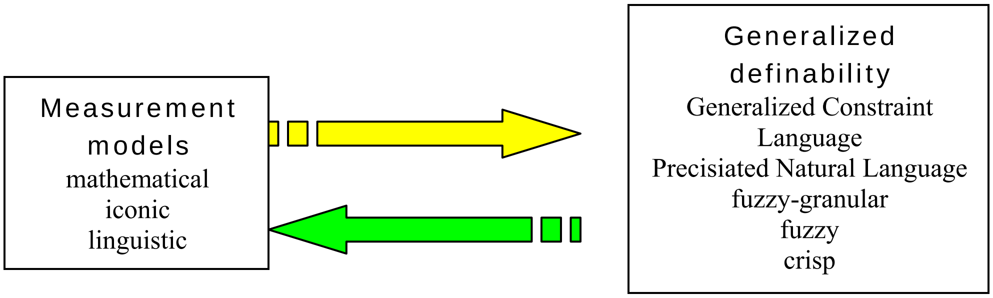

Figure 1. Mapping between model's hierarchy applied in measurement and generalized definability hierarchy

The relationship between the model classification used in measurement procedures and the definability hierarchy needs further study. Obviously, the fusion of various models used in measurement practice into heterogeneous compositions might produce better measurement results in different applications. Until computerization of a measurement technology, general measurement procedures had involved explicit quantities such as distance or voltage, determined by reading a scale or a meter [19]. Recent developments in adjacent areas have provided the technological support for measurements of more complicated systems, involving various implicit parameters, not directly accessible. Over last few years sensor technology has been undergoing revolutionary changes promising to have significant impact on a broad range of applications related to national security, health care, the environment, energy, food safety, and manufacturing. Sensor networks, which converge the Internet, communications, and information technologies with miniaturization techniques have provided new opportunities for acquiring and communicating huge arrays of information coming from heterogeneous sensors and sensor arrays. Communication networks offer rapid access to information and computing, eliminating the barriers of distance and time. The coming years will likely see a growing reliance on and need for more powerful sensor systems with increased performance and functionality. The new approach to measurement description involves data acquisition from whatever points are physically accessible following up with an estimation of the parameters of the system under measurement that is represented by its model [19].

This situation is typical for many traditional scientific applications. For example, in geological and geophysical sciences in the past fifteen years earthquake studies have grown from the collection of seismic data on frequency-limited seismometers (often in analog form) and mapping of surface faulting using geodetic techniques, to routine collection of digital seismic data on seismometers with broad frequency responses and mapping of surface deformation using a combination of geodetic, GPS and radar interferometry. In novel applications related to measurements in cyberspace the necessity of fusion of different models becomes even more important. In applications related to computer security and software quality one has to deal with the measurements inside very complex, unbounded and usually not strictly defined systems. For example, the number of possible metrics included in the guidelines for security evaluation [7] exceeds a few dozen. Some of the attributes used to calculate the corresponding metrics could be obtained from software and other sensors and counters while others are to be provided by the experts.

Measurement procedures actually implement modifications of the system models and adjustment of their parameters. Historically, they relied mainly on probability and statistics formalisms for their implementation. *This paper aims at demonstrating feasibility of fuzzy models application along with stochastic and statistical models for implementing such fundamental procedures of scientific cognition as measurement and estimating the possible gain we can achieve from its realization. Its goal is three-fold. It attempts:*

a) to develop a general method allowing fusing the prediction models of the values of the measured variables and their linear combinations with the measurement observations received from a real system,

b) to evaluate the accuracy of the measurements obtained with the prediction use and the possible gain, which a prediction application can give us,

c) *to derive conditions when a prediction application might produce the strongest benefits and to develop practical recommendations on how to achieve it in applications .*

Section II reviews and classifies the reported applications of fuzzy logic and other soft computing techniques in measurement procedures and sensor systems. Section III analyzes the epistemology of information sources used to produce prediction models, which can be applied in measurement procedures and their characteristics and investigates feasibility of fuzzy formalism application for their description. In section IV the prediction model is formalized as fuzzy. The problem of its application along with the measurement results is formulated as an optimization problem with fuzzy constraints. The properties of this problem solution are examined in the next section V. Section VI investigates the conditions of maximizing the gain achieved by fuzzy prediction models application in measurement and provides practical recommendations on how to attain it in complex networked sensor systems.

II. Soft computing models in measurement information fusion

During the last few years one can witness an explosive rise of publications reporting soft computing methodologies applications in measurement that are based upon some fuzzy and neural techniques. The classification given below does not pretend to be complete or comprehensive. It provides examples of the soft computing applications in multi-source measurement information fusion and multi-sensor system design according to the goals achieved with their introduction.

1. Fuzzy logic and neural network approaches to multi-source data association and fusion in measurement. This group actually refers to multi-sensor systems and sensor arrays [28]. Within the group, one can find synergetic systems (mainly software, but some hardware could be included as in [4]), which intend to provide universally applicable solutions for fusion of the signals from different sources or for decision-making based on a variety of measurement results. Bloch [3] gives a classification of numerical fusion operators which are applied to fuse imprecise, uncertain and incomplete information extracted from a variety of sensors with a degree of belief associated with each information source.

In [18] an extra knowledge is applied for choosing between different decisions made by solving conventional problems. In [17] the comprehensive methodology called Extended Logical Sensor Architecture (ELSA) was developed for constructing industrial sensor integration systems. ELSA has been developed for industrial applications, particularly, on-line grading and classification of non-uniform food products. The sensor design is based upon the object model, which represents object classifications through combinations of primary features weighted by fuzzy variables. The features guide the selection of sensors and processing routines; the classifications determine the rule base used by the inference engine for process decisions. Although inspection was the focus of this design, it is intended to become applicable in a variety of automation tasks, which may benefit from a multi-source perception system.

Another multi-source fusion technique called Recurrent Fuzzy Inference (RFI) is presented in [9]. Here the membership functions of RFI are expressed by Radial Basis Function neural networks with insensitive ranges. The shape of the membership functions can be adjusted by a learning algorithm. This algorithm is based on the steepest descent method and incremental learning, which can add new fuzzy rules.

A very interesting subsection of this group is composed by the multi-sensor systems attempting to reproduce human capabilities of taste, odor and vision analysis. Besides improving accuracy, convenience and efficiency, the method allows realizing an engineering model of the human's sensing and recognizing systems that combines all features of artificial olfactory and artificial taste. Rong et.al. [26] develop "electronic nose' and "electronic tongue" for wine classification based on fuzzy logic fusion technique. Llobet et.al. [13] describe an electronic nose based system, which employs an array of inexpensive commercial tin-oxide odor sensors, which have been used to analyze the state of ripeness of bananas. Readings were taken from the headspace of three sets of bananas during ripening over a period of 8-14 days. Lazzerini et.al. [11] present a new method for the fuzzy classification of odor samples that are obtained from an array of conducting polymer sensors. Linguistic expressions describing the response of both individual sensors and the sensor arrays to each chemical are derived from a fuzzy model of the sensor data.

2. *Fuzzy logic and neural networks application for pattern recognition and classification* represents one of the most advanced areas of fuzzy model applications.. Here soft computing techniques are applied to make recognition more reliable as in [12], where a smart eddy-current sensor for locating and identifying metal tags used to recognize buried pipes or more accurate as in [2]. However, those systems should not be compulsory multi-source.

3. *Fuzzy and neural methodology application for improving metrological and reliability characteristics*. In this group fuzzy and neural methodology is commonly applied alongside with some extra (sometimes a priori) information available [21,22]. Su and Komata [29] consider an in-vehicle-type load indicator and propose an error correction technique to compensate the error contained in the load measurement, by using fuzzy logic for dealing with changes in the loading states with a diversity of uncertainties. In [21] and [8] the system under measurement model, which could be presented with neuro-fuzzy methods is applied for a sensor fault detection and even correction. Healy et.al. [8] describe a sensor in-range fault accommodation, which is a fundamental challenge of dual channel control systems in modem aircraft gas turbine engines. An on-board, real-time engine model can be used to provide an analytical third sensor channel that may be used to detect and isolate sensor faults. A fuzzy-logic-based accommodation approach is proposed that enhances the effectiveness of the analytical third channel in the control system's fault isolation and accommodation scheme. In [22] method, the number of channels can be expanded and the sensor fault could be corrected. The similar approach [15] is applied for the validation of the measurement results in an ultrasonic sensor dedicated to mobile robot navigation

4. *Sensors, whose design is based on fuzzy neural network application*. Here one can find a sophisticated design, incorporating the advanced soft computing techniques based on fuzzy logic and neural networks application. In [5] and [10] two different networks, a feedforward neural network with an error backpropagation learning algorithm and a counterpropagation neural network, are employed to recognize the extracted features and provide a comparison of these two networks based on accuracy and speed. The data from multiple sensors are integrated through the proposed fuzzy logic model. Such a model is self-organizing and self-adjusting, learning from experience. Physical experiments of the metal cutting process are implemented to evaluate the proposed system.

Another example is [12], which develops a cascaded architecture of neural fuzzy networks with feature mapping (CNFM) to help the clustering of satellite sensor images. In the CNFM, a Kohonen's self-organizing feature map (SOFM) is used as a preprocessing layer for the reduction of a feature domain, which combines original multi-spectral gray values, structural measurements from co-occurrence matrices, and spectrum features from wavelet decomposition. In addition to the benefit of the feature space dimensional reduction, Kohonen's SOFM can remove some noisy areas and prevent the following training process from being overoriented to the training patterns, The condensed measurements are then forwarded into a neural fuzzy network, which performs supervised learning for pattern classification. The proposed cascaded approach is an appropriate technique for handling the classification problem in areas that exhibit large spatial variation and interclass heterogeneity (e.g., urban-rural infringing areas). The CNFM is a general and useful structure that can give us favorable results in terms of classification accuracy and learning speed.

In [33] a fuzzy c-means clustering algorithm is applied to classify training vectors, each cluster is trained by a radial basis function network. This approach is used to develop the so called soft sensor which is understood as the association of a sensor conducting actual measurements with an estimation of some other variables and model parameters. The soft sensor is applied to estimate a slab temperature in a practical walking beam reheating furniture.

III. Prediction Model Origins and Examples

The current philosophical viewpoint on measurement defines its task as a parameter adjustment of a system model formulated mentally (presumably by an expert), so it is able to predict the same results actually supplied by observation [19]. In complex engineering and science systems some information might come from sensors while another part may be supplied by the experts. Due to their nature, these sources differentiate in reliability and uncertainty of the information produced. Uncertainty also can be influenced by the characteristics of procedures and tools applied in information acquisition and processing. Experts may formulate their estimates based on their previous experience and general and domain knowledge. While lagging to instruments in accuracy of particular measurement results, the expert's estimates may complement them by providing information regarding relationships and associations between sensor signals that could be applied for improving overall accuracy and reliability.

Models of various kinds are used extensively to provide representations of some aspects of the real life system under measurement (see fig. 2). Among main types, linguistic and mathematical models are listed [31]. The linguistic model uses the natural language to express sufficient parameters and their interactions. In the empirical science and the arts this is the prime form for presenting models of situations and relationships. While it usually serves as the first level of modeling, it often plays a complimentary role, providing descriptions of parameters and is given by an expert. Imprecision of natural language is a major obstacle to application of conventional methods of computing with information decribed in natural language [40]. Fuzzy logic, according to its founder L.Zadeh, was specifically designed as a methodology for processing linguistic models the way that no other methodology can serve this purpose [38]. Starting with an introduction of the concepts of a linguistic variable and granulation [34] with further development of the theory of a fuzzy constraint and fuzzy constraint propagation [35-37] fuzzy logic has developed an all inclusive mechanism for expressing, formalization and processing of linguistic models, which may become an invaluable tool for formalizing initial information about the object or a process and its application in measurement procedures. Information, which we consider in this paper, represents a high level model of the underlying concepts of the objects or processes under measurement. The model could be provided or derived based upon either of the following or their combinations:

1. knowledge of the physical, biological, chemical, mechanical or other natural laws, according to which an underlying system or process operates,
2. knowledge of the design and operational characteristics of the sensors, communication networks and other equipment,
3. expert estimates,
4. data mining and knowledge acquisition methods including
    a. statistical methods based on regression analysis and other techniques,

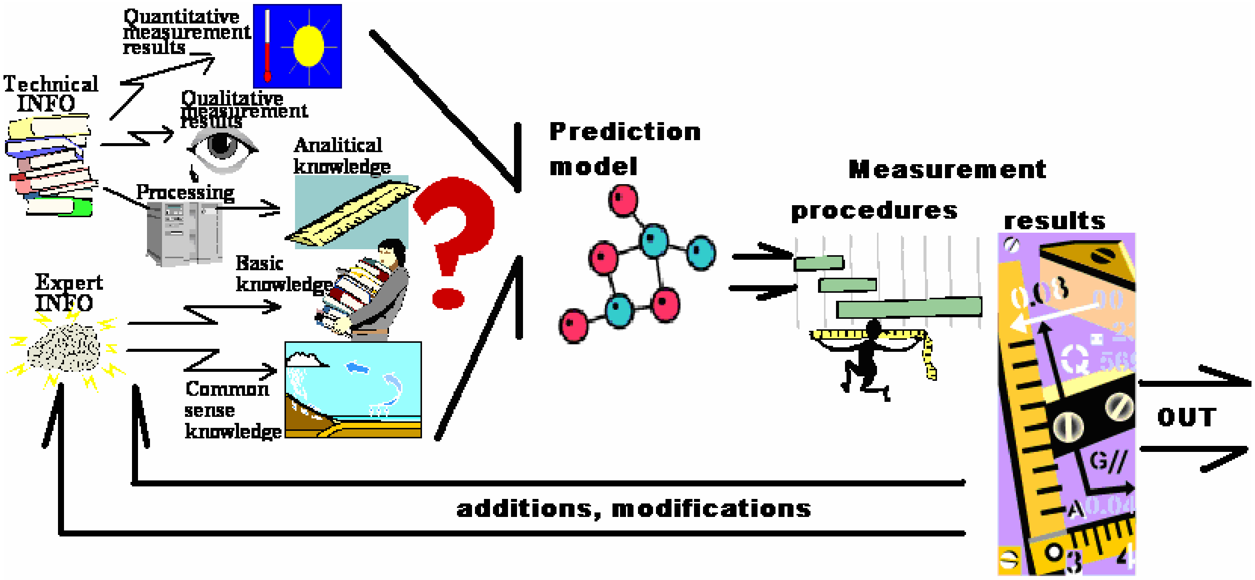

Figure 2. Measurement as a process of scientific cognition with various information sources

b. intelligent data-driven methodologies, such as fuzzy logic, neural networks, genetic algorithms.

This model could be expressed in a form of functional or order relationships, may be approximate, stochastic or fuzzy. The following examples of possible functional relationships and their applications may be given:

1. in the measurement of flow rates in a variety of pipelines, which converge into one we have $Q_1 + Q_2 + \ldots + Q_n = Q$, where $Q_1, Q_2, \ldots, Q_n$ are the measurement results of the flow rates in converging pipelines and Q is the measured flow rate in the common pipeline.
2. measurement techniques applied in the fault tolerant skewed inertial measurement units, which are currently used in many aircraft and space systems [4], in which parity residuals indicative of the sensor errors are derived and then compared to calculated thresholds.

Over the last few years new applications demonstrating more complicated schemes of using fuzzy formalized expert information in measurement procedures have been published. [14] describes an intelligent fusion measurement system in which measurement data from different types of sensors with various resolutions are integrated and fused based on the confidence in them derived from information not usually used in data fusion, such as operating temperature, frequency range, fatigue cycles, etc. These are fed as additional inputs to a fuzzy inference system (FIS) that has predefined membership functions for each of these variables. The output of the FIS are weights that are assigned to the different sensor measurement data that reflect the confidence in the sensor's behavior and performance. In [1] the fuzzy and interval analysis models of a two-dimensional navigation map and rough estimates of a robot position are applied to improve a robot guidance and navigation. [16] aims at reproducing the linguistic evaluation of comfort perception provided by a human through aggregating of the relevant physical measurements such as temperature, humidity, and luminosity.

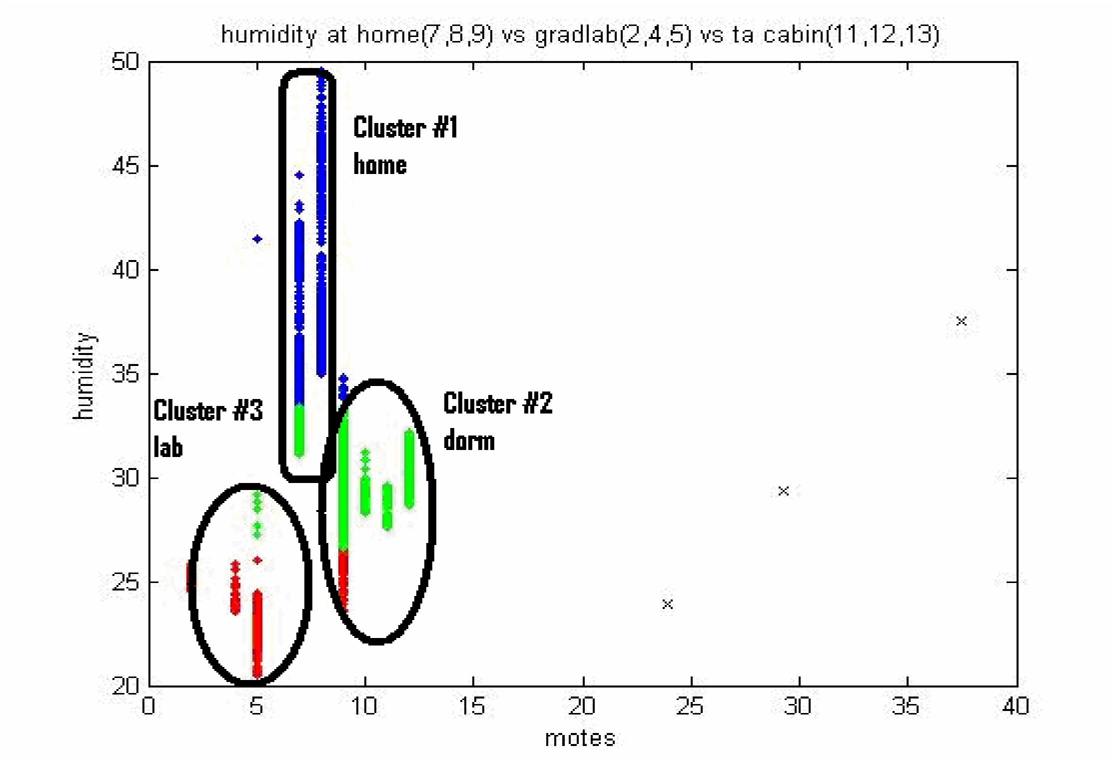

Figure 3. Measurements of humidity taken at by sensor nodes at different locations over a period of two days.

At the same time new emerging technology of wireless sensor networks (WSN) may provide a technological base for generation and communication of additional models. WSN are composed by many cheap sensor nodes, each of which usually includes a few sensors that may communicate and transmit information between each other and with other processors called base stations. In order to accomplish this task sensor nodes are supposed to be deployed in close proximity to each other. The energy supply of the sensor nodes is one of the main constraints of the technology, which needs to be considered in the design of WSN. One of the techniques widely used to reduce the energy consumption is implemented by grouping a number of nodes into a cluster and restricting outside communications between clusters and base stations. Due to the short distances between sensor nodes within the cluster, their measurement results are expected to be associated with each other as well. However, this association may be not strict and allows for variations and deviations due to a number of reasons, with the sensors low accuracy being probably the most contributing one. Figure 3 demonstrates the measurement results of humidity taken over a period of a few days in three locations under controlled environment: a home room, a dorm room and a college lab. The measurements were taken with Telos ver.B sensor nodes produced by Crossbow Inc., then they were communicated to, recorded and processed at the PC base station. One can see a clear association between measurement results produced by sensors positioned in a close proximity to each other. This kind of association could be used for formulating a prediction model. However, figure 3 demonstrates that the results are close but do not exactly repeat themselves. It means that the model incorporating a certain degree of uncertainty need to be applied. Fuzzy technique seems to be perfect for formalizing this sort of knowledge.

IV. Mathematical problem formulation

The general measurement goal as formulated above is to find estimates of the parameter vector X of some available system model. Also we assume that some prediction model regarding those parameter values is available. The model predicts the values of some functions f(X) of the parameter values. The functions f() are introduced to generalize a case where the prediction could be made about some association of a few parameters. The prediction is usually expressed as a model of type “the fuction f of the parameter vector X is R”, where R is a constraining relation. One can see that this prediction belongs to the class of a generalized constraint as determined by L.Zadeh [40]. This model could be formalized as a fuzzy model and described with the set of membership functions $\mu(f(X))$. A conventional way of finding the model parameter estimates based on the measurements would go through its definition as a mathematical programming problem and search for the parameter X' estimates by maximizing some criteria

(1) $$\hat{X} = \max_{x} F(Y_1, Y_2, ..., Y_n, X),$$

where F( ) is a functional, whose shape is determined by the estimation methods,

Yi (i=1,n) is a set of m_i measurements of the ith variable.

Let us consider a prediction model information as a fuzzy constraint for the parameter vector X and given by the set of membership functions μ(f(X)). The methods of a model's information acquisition and its propagation can be found in a number of publications (see, e.g. [16,23,32]). In this case the estimation problem with a prediction model application can be considered as an optimization problem with fuzzy constraints. By now research of fuzzy constraints (see [20] for one application) has developed a variety of methodologies of solving such problems. One of the simplest and the most obvious way is merging of the functional criteria and the constraints into one synergetic criterion and search for a global solution by the way of this criterion optimization. The problem can be re-formulated as search for the estimate maximizing the combination criterion

(2) $$\tilde{X} = \max_{X} F(Y1, Y2, \ldots, Yn, X) \times \mu(f(X))$$

This problem could be tried with conventional or intelligent methods. The method choice should depend on the estimation techniques applied as well as on the membership function shapes (see [24,25] for more detail). We will call the solution of this optimization problem a modified estimate.

V. Investigation of the Modified Estimates in Comparison to the Conventional Ones

A. Properties

Let us assume the probabilistic model of normal distribution, which is the most widely applied in practice for measurement results (equation (3)) and the prediction model, in which the approximate values of linear combinations of a few variables are given (equation (4)), which is again the mostly anticipated feasibility in a case of sensor networks:

(1) $Y = AX + \varepsilon y$

(2) $b \approx BX$

where Y is a n×1 vector (under the condition of n > 1) of measurement results,

X is a k×1 vector (under the condition of k > 1) of true values of the measurable variables,

εy is a n×1 vector (under the condition of n > 1) of measurement errors,

b is is a m×1 vector (under the condition of m > 1) of the prediction values,

A, B are matrices giving the structures of measurement schemes and prediction models.

(3) is a classical measurement equation applied in measurement theory and standards. We will consider measurement results normally distributed with no bias and the covariance matrix Σy.

(4) describes the prediction made that m linear combinations of the measured variables (the combinations are given with the matrix B) approximately have values given by the vector b components. The prediction is described mathematically by using the membership functions of the type (5) with the parameters of fuzziness given with the matrix Σb, which is a diagonal matrix with elements calculated as squares of the prediction fuzziness parameters.

Within the framework given above the following statements describing the role of a fuzzy prediction model in measurement are formulated. They will be proved under specified conditions in the section below.

Theorem 1. The modified measurement estimate that takes into account a fuzzy prediction model along with measurement results coincides with a conventional one that does not, if and only if the prediction value coincides with the conventional estimate.

Corollary 1. An application of fuzzy prediction models shifts the measurement result.

Theorem 2. The modified estimate sits between the conventional estimate and the prediction value.

Corollary 2. An application of fuzzy prediction models shifts the measurement result towards the prediction value.

With the direct measurements and predictions, matrices A and B become unit matrices and the equations (3) and (4) become simpler:

$Y = X + \varepsilon y$

$X \approx b$ or $x_1 \approx b_1, x_2 \approx b_2, \ldots, x_k \approx b_k$

meaning that the model predicts approximate values of all k measured variables. This type of prediction could be formalized with the membership function of type (5)

(5) $\mu(x) = \exp(-(x-b)^2 / \sigma^2)$,

where b gives the prediction value and σ is a parameter describing the fuzziness of prediction.

We are ready now to attack optimization problems (1) and (2) formulated in section IV. We will be looking for a solution maximizing the likelihood function, which is composed from normal distribution probability functions and in a case of a prediction model the fuzzy model membership function is added. Under general conditions, a conventional maximum likelihood estimate, which does not take into account a fuzzy prediction model and is based only on measurement results could be found as a solution of the optimization problem (1) as

$\hat{X} = (A^T \Sigma_y^{-1} A)^{-1} A^T \Sigma_y^{-1} Y$;

The modified estimate, which does take into account a fuzzy prediction model along with measurement results could be found as a solution of the optimization problem (2) as

$\widetilde{X} = (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1} (A^T \Sigma_y^{-1} Y + 2B^T \Sigma_b^{-1} b)$;

or in one variable case when we measure and predict just one variable

$\hat{X} = Y$; and

$\widetilde{X} = (Y/\sigma^2 + 2b/\delta^2)/(1/\sigma^2 + 2/\delta^2) = (Y + b/g^2)/(1 + 2/g^2)$, where $g = \delta / \sigma$ is the ratio of prediction uncertainty to measurement error, which is called later a prediction uncertainty factor.

The bias and the generalized dispersion of these estimates are equal correspondingly:

(6) $M(\hat{X} - X) = 0$; $\operatorname{cov}(\hat{X}) = M[(\hat{X} - M\hat{X})(\hat{X} - M\hat{X})^T] = (A^T \Sigma_y^{-1} A)^{-1}$

(7) $M(\tilde{X} - X) = 2(A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1} B^T \Sigma_b^{-1} (b - BX)$ (4)

$\operatorname{cov}(\tilde{X}) = M[(\tilde{X} - M\tilde{X})(\tilde{X} - M\tilde{X})^T] = (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1} A^T \Sigma_y^{-1} A (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)$

where M() serves as a mean operator.

The dependence of a modified estimate's bias on the prediction uncertainty factor under different prediction errors in the case of one measured variable and one prediction made is given in fig.4. The enlarged section of this graph, which demonstrates the region where the bias becomes comparable or less than a measurement error is given in fig. 5. One may see that the bias becomes pretty small when the prediction error is still about ten times higher than the prediction uncertainty. It means that the prediction model should be able to make reliable, practically unbiased predictions.

B. Bias of the modified estimates

The properties characterizing the accuracy of the modified estimate need to be investigated in order to evaluate a possible gain/lose.

Property 1. Analyzing the formulae above one may see that a modified estimate's bias is proportional to the prediction error (b-BX)

Property 2. One also can see that when a prediction is absolutely correct (b-BX=0) the modified estimate becomes unbiased.

*Property 3.*On the other hand, the same result can be achieved when the prediction fuzziness is very big.

The modified estimate becomes unbiased when the prediction model gives a correct prediction or refuses to make any prediction at all. Actually, the bias of the modified estimate mainly depends on the ratio between the prediction error and the prediction fuzziness parameter or in other words on the ratio between the prediction model correctness and the forecaster confidence in the prediction made.

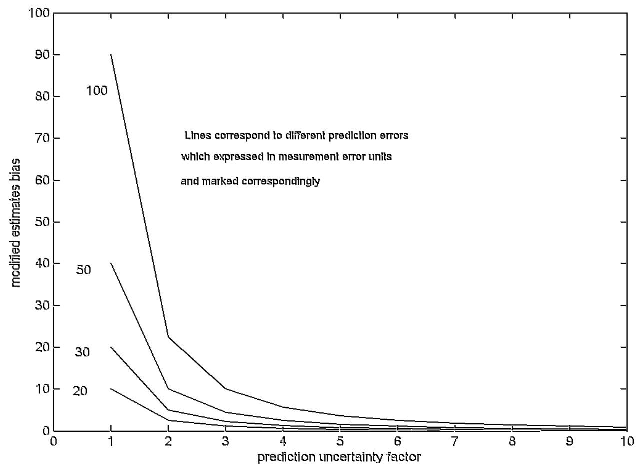

Figure 4. Relationship between the modified estimate's bias and the prediction uncertainty factor and prediction errors

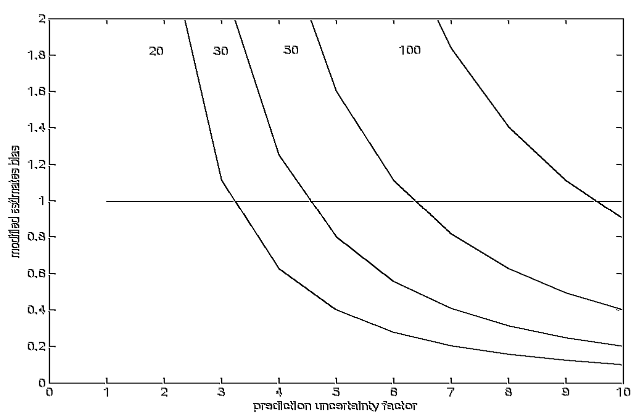

Figure 5. Relationship between the modified estimate's bias and the prediction uncertainty factor (enlarged section). The bold horizontal line marks the border where a bias becomes equal to a measurement error.

C. Efficiency of the modified estimates

Estimate's accuracy traditionally is taken as its dispersion. Comparing values (6) and (7) one may conclude that

Property 4. When the prediction fuzziness is very big (practically the prediction is not given) the dispersions of estimates almost coincide with each other, *Property 5.* Generally speaking the modified estimate's dispersion is smaller than the conventional one's that is formally making the modified estimate more efficient than the conventional one. It can be explained by the fact that the modified estimate is "pulled over" towards the prediction model value.

However, despite this pleasant result, the dispersion can not be taken as a good comparison base as the modified estimate could be biased due to the wrong prediction.

We have to consider another accuracy indicator, the mean square error (MSE), which is the mean of squares of deviations between the estimate and the true value. This indicator takes into account both the estimate's bias and its deviation from it. MSE of the considered estimates will equal correspondingly:

$$E_{\hat{X}} = M[(\hat{X} - X)(\hat{X} - X)^T] = (A^T \Sigma_y^{-1} A)^{-1};$$
$$E_{\tilde{X}} = M[(\tilde{X} - X)(\tilde{X} - X)^T] = (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1} (4B^T \Sigma_b^{-1} (b - BX)(b - BX)^T \Sigma_b^{-1} B +$$
$$+ A^T \Sigma_y^{-1} A)(A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1};$$

The problem of this gain evaluation deserves a special consideration. To evaluate the gain provided by an expert's information application, let us choose the projection of the estimate's MSE, which can be written as

$$T = \frac{1}{K} Sp(E_{\hat{X}} E_{\tilde{X}}^{-1}) =$$

$$= \frac{1}{K} Sp\ [\ (A^T \Sigma_y^{-1} A)^{-1} (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)(4B^T \Sigma_b^{-1} (b - BX)(b - BX)^T \Sigma_b^{-1} B + A^T \Sigma_y^{-1} A)^{-1}\ (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)]$$

The gain depends on the measurement system structure and errors as well as prediction model errors and its fuzziness indicator (actually the ratio between the prediction error to the fuzziness). Figure 6 demonstrates the change in the gain value depending on the uncertainty prediction factor or actually on the prediction fuzziness when the mean measurement error is fixed. The maximum gain could be achieved when the prediction is absolutely accurate (prediction error is zero), that corresponds to the top line in fig 6. Other lines show the relationship under the condition of some prediction error. This dependence on the prediction error is clearer in Figure 7, which demonstrates the gain change depending on it under different uncertainty prediction factors. One can see that with the increase in prediction errors the gain goes down and transforms into lose when the error in prediction becomes considerably bigger than its fuzziness.

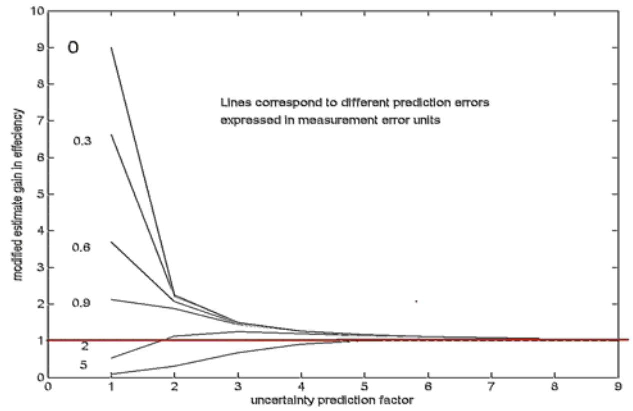

Figure 6. Relationship between the gain received by a modified estimate application and uncertainty prediction factor and prediction errors

In a case of one measured value and one prediction the gain can be expressed as

$$T = \sigma^2 (1/\sigma^2 + 2/\delta^2)^2 /(4(b-x)^2 / g^4\sigma^4 + 1/\sigma^2)$$

or in terms of a prediction uncertainty factor

$$T = \sigma^2 (g^2 + 2)^2 /(4(b-x)^2 + g^4\sigma^2)$$

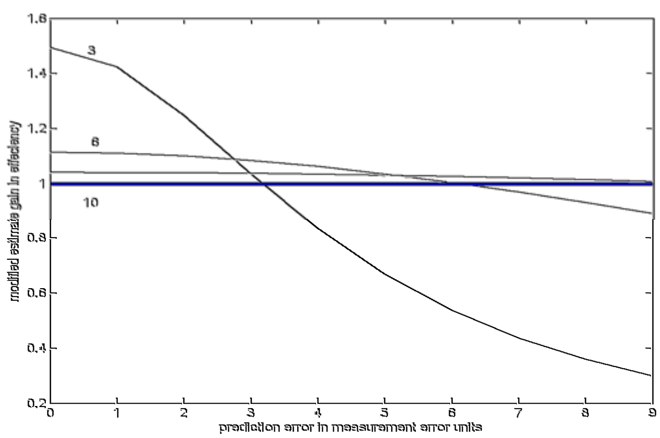

Figure 7. Relationship between the gain received by a modified estimate application and prediction errors and uncertainty prediction factor. Note: everything is scaled in measurement error units

The biggest gain can be achieved with the correct predictions, i.e. b=BX, when

$$T_{\max} = Sp[(U + 2(A^T \Sigma_y^{-1} A)^{-1} B^T \Sigma_b^{-1} B)^2],$$

where U is a unit matrix or in the case of direct measurements and predictions

$$T_{\max} = Sp[(U + 2(\Sigma_y \Sigma_b^{-1})^2],$$

from where one may see that the maximum gain value depends on the ratio of the measurement errors to the prediction fuzziness. In another notation the maximum gain could be written as

$T_{\max} = (1 + 2\sum_{i=1}^{K} \sigma_i / \sum_{i=1}^{K} \delta_i)^2$, which in the case of one measured variable only becomes

$$T_{\max} = (1 + 2\sigma / \delta)^2.$$

One may see that in order to increase the maximum gain, the prediction fuzziness should be decreased in comparison with the measurement errors. One may also conclude that if the prediction fuzziness is higher than 100 times measurement errors, the use of a prediction model becomes doubtful as any possible gain value could be just a few percent. It means that one has to try to develop prediction models with a lower prediction fuzziness parameter. However, this strategy might be risky as it may result in loosing any gain at all.

VI. WHAT CAN WE GAIN FROM USING FUZZY PREDICTION MODELS AND UNDER WHICH CONDITIONS?

A. Mathematical point of view

Theorem 3. Fuzzy prediction model application results in accuracy gain if the prediction errors are smaller than the weighted sum of measurement errors and prediction fuzziness – see (8) and (9)

Corollary 3. If the prediction errors are smaller than the prediction model fuzziness, than definitely the prediction model application will result in accuracy gain.

Let us try to clarify conditions when the modified estimate superiors a conventional one against the MSE indicator or becomes more accurate and to prove the theorem given above. Mathematically the condition

$E_{\tilde{X}} < E_{\hat{X}}$ can be shown equivalent to the condition

$B^T \Sigma_b^{-1}(b - BX)(b - BX)^T \Sigma_b^{-1} B < B^T \Sigma_b^{-1} B + B^T \Sigma_b^{-1} B(A^T \Sigma_y^{-1} A)^{-1} B^T \Sigma_b^{-1} B$;

or in other notation

(8) $(b - BX)(b - BX)^T < \Sigma_b + B(A^T \Sigma_y^{-1} A)^{-1} B^T$.

The left side of this inequality constitutes the prediction error square, the right side combines the prediction fuzziness with measurement errors and due to the structures of matrices Σ_y and Σ_b in squares also. One may see that in order to increase the estimate accuracy, the prediction error should be not bigger than its fuzziness. Actually, it may be even larger by some value, which depends on the measurement errors. This relationship becomes clearer in the case of direct measurements and predictions when the matrices A and B are unit matrices, and matrices Σ_b and Σy are diagonals. In this case the condition (8) becomes more transparent as

(9) $\sum_{i=1}^{K}(b_i - x_i)^2 / K < \sum_{i=1}^{K}\delta_i^2 / K + \sum_{i=1}^{K}\sigma_i^2 / K$

where K is the number of measured variables,

σi, i=1, K is the root mean square error (RMSE) of the i-th variable measurement errors, and δi, i=1, K is the fuzziness of the ith prediction.

Use of a priori information improves accuracy if the mean prediction error is less than the sum of the mean prediction fuzziness and the mean measurement error.

B. *Practical point of view*

Or in other words what gain could be achieved with a rather inaccurate prediction?

The typical measurement accuracy for the modern measurement instruments could be in the vicinity of 1-2%. In this case, of say 2% measurement error, with the prediction fuzziness of say 20%, which is rather high (for example, it might mean the prediction like “the measured variable has a value of around

10 units or actually somewhere roughly between 8 and 12 units”, which in practical cases sounds like a very reasonable suggestion, could achieve the gain up to 44%.

In complex measurement systems exploiting sensor networks, accuracy is actually much lower. In a case of around 10% measurement errors, the same predictions as in the previous example could achieve 400% gain. One should understand that such values of gain could be achieved when the prediction model makes an absolutely correct prediction.

However, even in a case when a prediction model includes a prediction error, there could still be some gain. Speaking very roughly, in order to get any gain the error value should be smaller than a sum of the prediction fuzziness and the measurement errors. This relationship allows a forecaster to develop a strategy to avoid any loss. If a forecaster is confident about the prediction value, a fuzzy prediction model with low prediction fuzziness should be applied and a high gain could be achieved. However, when the confidence level decreases, the prediction fuzziness could be increased that might make the gain value lower but allows avoiding losses.

VII. Conclusion

Measurement as an underlying concept of scientific cognition is the process of formulating and modifying models of the systems under measurement and the environment based on observation results received. Different model types are applied in measurement. Their relationship and their place in the model hierarchy introduced by L.Zadeh in his theory of generalized definability [40] as well as the benefits we can get from their application call for a further study.

The problem of a fuzzy prediction model use for improvement the measurement procedures quality and the accuracy of received estimates may be solved by fusion of information from different sources, which are characterized by different degrees of uncertainty. The problem has been formalized mathematically as an optimization problem with fuzzy constraints and the solution has been found for the normally distributed measurement results and a specific expert’s information.

The properties of the modified estimates have been investigated in comparison with the conventional ones. The modified estimates have been found more efficient under the condition when the prediction error does not overcome the sum of the average measurement error and the prediction fuzziness. The possible efficiency gain in practical applications was estimated. The procedures improving reliability of the modified estimates have been offered, which include recommendations on formulating prediction models: when the confidence in predicted value is high, a prediction fuzziness should be made low in order to achieve a high gain; however, with a decrease in confidence a fuzziness could be made wider in order to avoid the estimate corruption.

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