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Traffic Assignment for Risk Averse Drivers in a Stochastic Network

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Abstract: Most traffic assignment tasks in practice are performed by using deterministic network (DN) models, which assume that the link travel time is uniquely determined by a link performance function. In reality, link travel time, at a given link volume, is a random variable. Such stochastic network (SN) models are not widely used because the traffic assignment algorithms are much more computationally complex and difficult to understand by practitioners. In this paper, we derive an equivalent link disutility (ELD) function, for the case of risk averse drivers in a SN, without assuming any distribution of link travel time. We further derive a simpler form of the ELD function in a SN which can be easily implemented in deterministic user equilibrium traffic assignment algorithms like a DN. By comparing our two derived ELD functions, the bound of the coefficient of the simpler ELD functions is obtained, so that drivers will make the same risk averse route choice decisions. A method to estimate the coefficient of the simpler ELD function has been proposed and demonstrated with questionnaire survey data gathered in El Paso, Texas. The results of user equilibrium traffic assignments in a test network using the Bureau of Public Roads (BPR) function and the simpler ELD function are then compared. Our simpler ELD function provides a mean for practitioners to use deterministic user equilibrium traffic assignment algorithms to solve the traffic assignment problem in a SN for risk averse drivers during the peak hour commute.

BACKGROUND AND MOTIVATION

At present, most traffic assignment models used in practice assume that travel time in a link is a deterministic function of the link's characteristics (such as free-flow travel time and link capacity) and link volume. A network with such a deterministic link travel time function is called a Deterministic Network (DN) *(1)*. In reality, for the same volume in a link, we have variations in travel time. These variations are due to the differences in vehicle composition, lane distribution, driving behavior, weather, incidents, etc. These variations are usually small when the volume is light but they become larger as the link becomes more congested. One way to model such variation is to consider link travel time as a random variable that has a probability distribution, with mean and variance expressed as functions of the link characteristics and link volume. A network with such probabilistic link travel times is called a Stochastic Network (SN) *(1)*.

Most transportation network models assume that (1) the drivers have perfect knowledge of the link travel times (in a DN) or of the probabilities of different values of link travel times (in a SN); and (2) the drivers will select the routes that will minimizes the travel times between their origins and destinations. The resulting state of the transportation network is called Deterministic User Equilibrium (DUE). In reality, a driver's knowledge is usually somewhat imperfect. The driver's perception of a link travel time may be slightly different from the actual travel time. Some transportation network models take this perception error into account by modeling it as a normal distribution with zero mean. Due to these perception errors the selected routes of the drivers vary stochastically. The resulting state of the transportation network is called Stochastic User Equilibrium (SUE) *(1)*.

Based on the assumptions in the nature of link travel times and drivers' perception on the link travel times, traffic assignment models may therefore be classified into four types: Deterministic Network-Deterministic User Equilibrium (DN-DUE), Deterministic Network-Stochastic User Equilibrium (DN-SUE), Stochastic Network-Deterministic User Equilibrium (SN-DUE), Stochastic Network-Stochastic User Equilibrium (SN-SUE) *(1,2)*.

The DN-DUE is the simplest, the easiest to understand, and the most widely used traffic assignment model in practice. This model was originally formulated by Beckman et al. *(3)* and may be solved by DUE algorithms (for examples, the Frank-Wolfe algorithm *(4)*, Algorithm B *(5),* and others). In DN-SUE models, the network's link travel times are deterministic (with a given volume distribution), but they may be perceived differently by different drivers. Due to the error in travel time perception, drivers will always select what they perceive as the shortest paths but these may not be the actual shortest paths. The DN-SUE model was originally formulated by Daganzo and Sheffi *(6)*. A popular solution algorithm for the DN-SUE model is the Method of Successive Averages (MSA) *(7)*.

In a SN, driver's response to travel time uncertainty has also been modeled. Instead of selecting the route which has the minimum expected travel time, the driver is modeled to select the route that has the minimum expected disutility. Such SN model was first studied by Mirchandani and Soroush *(8)*. While DN-DUE and DN-SUE models are used by many transportation modelers, only few papers such as *(1, 2, 8, 9, 10)* used SN models because these models are much more computationally complex than the DN models. Under certain conditions, the SN-DUE model can be solved by DUE algorithms simply by replacing the link travel time function with a suitable equivalent link disutility (ELD) function *(8, 11)*.

In principle, it is possible to consider an even more realistic SN-SUE model which adds drivers' perception errors into the link travel time variations. However, according to *(2),* the SN-

DUE model is suitable for modeling of peak hour traffic because regular commuters have a good knowledge of the mean and variance of peak hour travel times in familiar routes.

To use the SN-DUE model to assign traffic for the peak hour commute in a network, a simple ELD function has been derived in this research. This function takes into account the link characteristics and driver's respond to uncertainty in link travel time, but unlike in (8, *9, 11*), it does not require the modeler to specify the link travel time distribution or variance. With this ELD function, a modeler solves the SN-DUE model as a DN-DUE model simply by replacing the deterministic link travel time function in a DN with the ELD function in a SN, and then applying the DUE algorithm.

The outline of this paper is as follows. After reviewing the previous related work, we proceed to derive the ELD function from route disutility function for drivers with risk averse route choice behavior, without assuming any probability distribution for the link travel time. Following this, the next section derives the simple ELD function without assuming the link travel time distribution, variance and driver's risk taking behavior. By comparing the two derived ELD functions, the constraint in the coefficient of the simple ELD function for risk averse drivers is obtained. We then describe a method to estimate this coefficient and demonstrate it with survey data gathered in El Paso, Texas. Using the coefficient obtained from the survey, the simple ELD function is implemented in a SN-DUE model for a test network, and the results is compared with that obtained in the DN-DUE model.

REVIOUS RELATED WORK

The Bureau of Public Road (BPR) function is the most popular function that describes the link travel time t_i in link i in a DN:

$$t_i = t_i^f \left[1 + \alpha \left(\frac{v_i}{c_i} \right)^\beta \right]$$ (1)

where t_i^f is the free-flow travel time in link i, v_i is the volume in link i, c_i is the capacity of link i, and α and β are constants. Typical values of α and β are 0.15 and 4 respectively.

To model a SN, the authors of (*9, 11*) modeled t_i as Gamma distribution with a lower bound equal to t_i^f. In a SN, it is reasonable to assume that (1) describes the average link travel time $\bar{t}_i$ in link i:

$$\bar{t}_i = t_i^f \left[1 + \alpha \left(\frac{v_i}{c_i} \right)^\beta \right]$$ (2)

In a SN, since link travel times are stochastic, the route travel times are stochastic. The driver's route selection depends on how the he/she react to the route travel time uncertainty. This is particularly important if he/she has constraint in the time of arrival (e.g., scheduled events, work starting times) with heavy penalties for late arrivals. There are three types of such behavior: risk averse, risk prone and risk neutral (*1, 2, 8, 9, 11*). The term risk here refers to the

risk of a late arrival at the destination. A risk averse driver prefers a route that has a longer average travel time but smaller variance to a route that has a faster average travel time but higher variance. That is, he/she would rather use the route with longer travel time (and depart early) to lower the risk of arriving late. On the contrary, a risk prone driver would select the route with a faster travel time but higher variation. A driver with risk neutral behavior does not consider travel time variation in his/her route choice decision. In the morning commute, it is reasonable to assume that majority of the drivers are risk averse.

According to decision theory (see for example (*12*)), in the stochastic case, a rational decision maker maximizes the expected value of his/her utility function, or equivalently minimizes the expected value of the disutility function. In a SN, given a choice of routes $r \in R$ connecting an origin-destination (O-D) pair, a driver will select the route r' which has the smallest expected route disutility $E[DU_r]$

$$E[DU_{r'}] = \min_{r \in R}\{E[DU_r]\} \tag{3}$$

For the drivers with risk neutral behavior, the route disutility DU_r is equal to the route travel time t_r. Therefore $E[DU_r]$ is equal to the average route travel time $\bar{t}_r$. For a route r which is made up of L links $t_r = t_1 + ... + t_L$. Therefore, $\bar{t}_r = \bar{t}_1 + ... + \bar{t}_L$. Thus (3) is equivalent to selecting a route with the smallest $\bar{t}_r = \bar{t}_1 + ... + \bar{t}_L$. Hence, a risk neutral driver uses the ELD function $DU_i = \bar{t}_i$ for route choice.

For describing risk averse and risk prone behavior, the most commonly used disutility functions are the exponential functions (*12*). Such functions have been used by (*1, 8, 9, 10, 11*):

$$DU_r = \begin{cases} b_1[exp(\omega t_r) - 1] & \text{for risk averse drivers} \\ b_2[1 - exp(-\varphi t_r)] & \text{for risk prone drivers} \end{cases} \tag{4}$$

where $b_1, b_2, \omega, \varphi$ are positive constants. According to (3), selecting a route with the smallest value of $E[DU_r]$ is equivalent to selecting a route with the smallest value of $du_r = \sum_{i \in r} DU_i$, the sum of the ELDs along the route.

In particular, in the SN-DUE case, when there is no perception error, for risk averse drivers the ELD function takes the following form (*11*)

$$DU_i = \bar{t}_i + c\sigma_{t_i}^2\left(\frac{1}{2} + \frac{1}{3}c\frac{\sigma_{t_i}^2}{\bar{t}_i - t_i^f}\right) \tag{5}$$

where $\sigma_{t_i}^2$ is the variance of the travel time of link i and c is a constant determined by the parameters of the exponential disutility function. It is important to note that, in order to use (5), one needs to know the $\sigma_{t_i}^2$ for every link.

The fact that the users preferences can be expressed in the form of minimizing the expression $du_r = \sum\limits_{i \in r} DU_i$ allows us to use a DUE solution algorithm to solve the SN-DUE model *(1, 2)*.

DERIVATION OF EQUIVALENT LINK DISUTILITY FUNCTION FROM ROUTE DISUTILITY FUNCTION

In this section, we derive an ELD function DU_i for risk averse drivers in a SN-DUE model, without the need to assume a link travel time distribution (e.g., Gamma distribution in *(9, 11)*).

According to the SN-DUE model, a driver selects a route that has the minimum value of $E[DU_r]$. Consider $A_r = g(E[DU_r])$ for some monotonically increasing function $g(x)$, minimizing $E[DU_r]$ is equivalent to minimizing A_r. For risk averse drivers, following (4), we can use

$$A_r = E[exp(\omega t_r)] \tag{6}$$

Since $t_r = t_1 + ... + t_L$, we can expand (6) to

$$A_r = E[exp(\omega(t_1 + t_2 + ... + t_L))] = E[exp(\omega t_1)exp(\omega t_2)...exp(\omega t_L)]$$

$$= E[exp(\omega t_1)] \cdot E[exp(\omega t_2)] \cdot ... \cdot E[exp(\omega t_L)] \tag{7}$$

Since $\frac{1}{\omega}ln(x)$, $\omega > 0$, is a monotonically increasing function of x, this choice of minimizes A_r is, in its turn, equivalent to minimizes $\frac{1}{\omega}ln(A_r)$. Here

$$\frac{1}{\omega}ln(A_r) = \frac{1}{\omega}ln\{E[exp(\omega t_1)]\} + \frac{1}{\omega}ln\{E[exp(\omega t_2)]\} + ... + \frac{1}{\omega}ln\{E[exp(\omega t_L)]\} \tag{8}$$

In a DN, in which all travel times t_i and t_r are deterministic, the above expression reduces to

$$\frac{1}{\omega}ln(A_r) = \frac{1}{\omega}ln[exp(\omega t_1)] + \frac{1}{\omega}ln[exp(\omega t_2)] + ... + \frac{1}{\omega}ln[exp(\omega t_L)] = (t_1 + t_2 + ... + t_L) = t_r \tag{9}$$

From Equation (8), we conclude that the new route utility function du_r can be expressed as $du_r = DU_1 + ... + DU_L$, where

$$DU_i = \frac{1}{\omega}ln\{E[exp(\omega t_i)]\} \tag{10}$$

Thus, the drivers preference in SN-DUE is equivalent to selecting a route with the smallest value of du_r. Therefore, selecting a route in a SN (for risk averse drivers) is very similar to selecting a route in a DN, but with link disutility DU_i instead of link travel time t_i.

Let us reformulate the expression for DU_i in terms of mean and variance of t_i. In a SN, the t_i in link *i* can be expressed:

$$t_i = \bar{t}_i + (t_i - \bar{t}_i)$$ (11)

It follows that

$$exp(\omega t_i) = exp(\omega \bar{t}_i) exp(\omega(t_i - \bar{t}_i))$$ (12)

Hence

$$E[exp(\omega t_i)] = exp(\omega \bar{t}_i) E[exp(\omega(t_i - \bar{t}_i))]$$ (13)

Usually $\omega(t_i - \bar{t}_i)$ is small, so we can expand the exponential function into the Taylor series and only keep the first three terms in this expansion

$$exp(\omega(t_i - \bar{t}_i)) \approx 1 + \omega(t_i - \bar{t}_i) + \frac{\omega^2 (t_i - \bar{t}_i)^2}{2}$$ (14)

Therefore

$$E[exp(\omega(t_i - \bar{t}_i))] = 1 + \omega E[t_i - \bar{t}_i] + \frac{\omega^2}{2} E[(t_i - \bar{t}_i)^2]$$ (15)

By definition, $E[t_i - \bar{t}_i] = 0$ and $E[(t_i - \bar{t}_i)^2] = \sigma_{t_i}^2$. Substituting (15) into (13), we obtain

$$E[exp(\omega t_i)] = exp(\omega \bar{t}_i)\left[1 + \frac{\omega^2}{2}\sigma_{t_i}^2\right]$$ (16)

The ELD thus becomes

$$DU_i = \frac{1}{\omega} ln\{E[exp(\omega t_i)]\} = \frac{1}{\omega} ln\left\{exp(\omega \bar{t}_i)\left[1 + \frac{\omega^2}{2}\sigma_{t_i}^2\right]\right\}$$

$$= \bar{t}_i + \frac{1}{\omega} ln\left[1 + \frac{\omega^2}{2}\sigma_{t_i}^2\right]$$ (17)

Using the Taylor series expansion of $ln(1 + z) = z + ...$ we obtain

$$DU_i = \bar{t}_i + \frac{\omega}{2}\sigma_{t_i}^2 \tag{18}$$

We have shown that, if the all drivers in a network have risk averse behavior, solving for DUE in a SN is similar to solving for DUE in a DN, except that we replace t_i in a DN with DU_i in a SN. The first term $\bar{t}_i$ in DU_i in (18) is taken from (2) which is essentially the same as the BPR function. Thus, it can be said that, in a SN, the additional term in the route choice decision for risk averse drivers is the link travel time variance, scaled by a factor $\omega/2$ ($\omega>0$). The magnitude of ω reflects the sensitivity of the drivers in avoiding the risk of late arrival. Risk averse drivers will avoid links that have large $\sigma_{t_i}^2$. If $\sigma_{t_i}=0$, the SN-DUE model is reduced to a DN-DUE model. Equations (5) and (18) have similar forms. In (5), t_i is assumed to follow Gamma distribution but in (18) we have not assumed any distribution for t_i.

ALTERNATIVE DERIVATION OF EQUIVALENT LINK DISUTILITY FUNCTION

This section presents an alternative derivation of a simpler ELD without assuming the distribution of t_i, $\sigma_{t_i}^2$ and driver's risk taking behavior.

In a SN, t_i is a random variable (at a given v_i), while $\bar{t}_i$ may be estimated by (2). Note that, in (2), when $v_i=0$, $\bar{t}_i = t_i^f$ which is deterministic.

It is natural to assume that DU_i depends on t_i^f and the relative average delay d, i.e.,

$$DU_i = F\left(t_i^f, d\right) \tag{19}$$

where

$$d = \frac{\bar{t}_i - t_i^f}{t_i^f} = \alpha\left(\frac{v_i}{c_i}\right)^\beta \tag{20}$$

for some function $F\left(t_i^f, d\right)$.

One would expect a link which has a higher t_i^f to have a higher link disutility; so, $F\left(t_i^f, d\right)$ must be an increasing function of t_i^f. One would also expect that as the link becomes more congested, t_i and hence the link disutility increases. In addition, field data have suggested that, as the link becomes more congested, the variation of link speed increases (see the speed-volume plots in (13)). Therefore, as the link becomes more congested, the variation of t_i and therefore the link disutility would increase; so, $F\left(t_i^f, d\right)$ must also be an increasing function of d. The function $F\left(t_i^f, d\right)$ must also satisfy the following conditions:

(i) When $v_i = 0$, $d = 0$, $t_i = \bar{t}_i = t_i^f$, therefore

$$F\left(t_i^f, 0\right) = t_i^f \tag{21}$$

(ii) If we sub-divide a link into a series of shorter links, the equivalent disutility of the original link must be equal to the sum of the equivalent disutilities of the shorter links. If we sub-divide a link (link *i*) into two sub-links (i_1, i_2) with free-flow travel times $t_{i_1}^f$ and $t_{i_2}^f$ respectively, then $v_{i_1} = v_{i_2} = v_i$, and $c_{i_1} = c_{i_2} = c_i$; so by (20), the *d* for both sub-links are the same as for the original link. Thus

$$F\left(t_{i_1}^f + t_{i_2}^f, d\right) = F\left(t_{i_1}^f, d\right) + F\left(t_{i_2}^f, d\right) \tag{22}$$

We then fix a value *d* and introduce an auxiliary function $G(a) = F(a, d)$. Equation (22) then takes the form

$$G(a+b) = G(a) + G(b) \tag{23}$$

We know that $F\left(t_i^f, d\right)$ is an increasing function of t_i^f and therefore, $G(a)$ is an increasing function of *a*. It is known that every monotonically increasing function $G(a)$ which satisfies (23) has the form $G(a) = k \cdot a$ for some *k*>0 *(14)*. For different *d*, the coefficient *k* may in general be different: $k = k(d)$. Thus we conclude that

$$DU_i = F\left(t_i^f, d\right) = t_i^f k(d) \tag{24}$$

From (21), we know that for *d*=0 we have $F\left(t_i^f, d\right) = t_i^f$. Therefore $k(0) = 1$.

For typical values of α and β, from (20) we have $d \ll 1$. Expanding $k(d)$ into a Taylor series, and ignoring the higher order terms, we have $k(d) = 1 + a_1 d + a_2 d^2$. Substituting the formula for into (24), we obtain

$$DU_i = t_i^f \left[1 + a_1 \alpha \left(\frac{v_i}{c_i}\right)^\beta + a_2 \alpha^2 \left(\frac{v_i}{c_i}\right)^{2\beta} \right] \tag{25}$$

Furthermore, for the standard values of $\alpha = 0.15$, $\beta = 4$, and the normal range of v_i/c_i, the term $\alpha^2 (v_i/c_i)^{2\beta}$ is usually negligible. Therefore we may simplify (25) as

$$DU_i = t_i^f \left[1 + a_1 \alpha \left(\frac{v_i}{c_i}\right)^\beta \right] \tag{26}$$

Equation (26) can also be expressed as

$$DU_i = \bar{t}_i + t_i^f \left[(a_1 - 1)\alpha \left(\frac{v_i}{c_i} \right)^\beta \right] \tag{27}$$

Hence, we may view DU_i as consisting of two components: the "deterministic" component $\bar{t}_i$ which has the same value given by the BPR function, and the "stochastic" component $t_i^f [...]$ which is due to the uncertainty in link travel time. Then, a_1 describes the sensitivity of the driver in respond to this uncertainty. We called a_1 risk averse coefficient in this paper. Note that, when $a_1 = 1$, drivers do not consider travel time uncertainty in route choice, and (27) is reduced to the BPR function.

Comparing (18) with (27), the latter is easier to implement in DUE algorithms as one does not need to know the $\sigma_{t_i}^2$ of every link. However, the condition of $\sigma_i^2 \geq 0$ imposes a restriction on the a_1 value. By equating the last term of (18) and (27), and with $\sigma_i^2 \geq 0$

$$\sigma_i^2 = \frac{2}{\omega} t_i^f (a_1 - 1)\alpha \left(\frac{v_i}{c_i} \right)^\beta \geq 0 \tag{28}$$

As all other terms in (28) are positive, it follows that $a_1 \geq 1$.

To use (27) in a DUE algorithm, one only needs to know the value of a_1. In principle, every driver should have his/her individual a_1 value. To describe the general behavior of the driving population, average value of a_1 may be used. The following section describes a method to estimate the average a_1 value from the results of a driver survey.

ESTIMATION OF RISK AVERSE COEFFICIENT

By expressing (26) in terms of d, we get

$$DU_i = t_i^f [1 + a_1 d] = t_i^f \left[1 + a_1 \left(\frac{\bar{t}_i - t_i^f}{t_i^f} \right) \right] \tag{29}$$

In a hypothetical link that has a constant travel time

$$DU_i = t_i^f \tag{30}$$

Suppose that there are only two parallel links connecting an O-D pair, with link $i=1$ having a constant travel time t_1^f, while link $i=2$ having a travel time according to (29). For link $i=2$, the values of t_2^f and $\bar{t}_2$ are prescribed as the minimum and average travel times respectively. Given the values of t_2^f, $\bar{t}_2$, we may ask a driver to specify the value of t_1^f such that they do not have any preference on one link over another. Under this condition

$$t_1^f = t_2^f \left[1 + a_1 \left(\frac{\bar{t}_2 - t_2^f}{t_2^f} \right) \right] \tag{31}$$

We may then solve for a_1.

A questionnaire survey has been conducted in the city of El Paso, Texas, to estimate the average a_1 value among the driving population. In this survey, participants were presented with the scenario of morning commute to work that has a fixed work-start time with a penalty for late arrival. There are two questions in the survey. Question 1 has t_2^f =20 minutes and $\bar{t}_2$ =30 minutes while Question 2 has t_2^f =35 minutes and $\bar{t}_2$ =50. In each of the questions, participants were given a set of possible t_1^f values at 5-minute increments. Each person was asked to select the closest t_1^f value in each question that satisfies (31), that is, he/she do not have preference between link 1 (which has a constant time t_1^f) and link 2 (which has an uncertain travel time). The two questions with different travel times were designed to check the consistency in the route choice behavior. They also help to find an average a_1 values for different trip lengths. The t_2^f, $\bar{t}_2$ values posed in the two questions are the typical ranges found in El Paso. Survey responds were collected from 202 drivers. There were 404 a_1 values computed from (31). The average values of a_1 is 1.4356. This indicates that an average driver is risk averse (since $a_1 > 1$) in the morning commute to work.

TEST NETWORK

In this section, a test network, adopted from (*3*), is used to illustrate the application of (26), the simple ELD, in a SN-DUE model (using the value of a_1=1.4356 obtained in the survey) and compare the results against the DN-DUE model.

The test network has been coded into TransCAD (*15*). The 25 nodes, 40 two-way links, t_i^f and one-way link capacity c_i are shown in Figure 1. The links with capacity of 300 vph have free-flow speeds of 20 mph while those links with capacity of 200 vph have free-flow speeds of 55 mph. Only nodes 7, 9, 17, 19 are O-D node. The O-D matrix is shown in Table 1. TransCAD uses the Frank-Wolfe algorithm to solve the DUE problem (*16*). After network coding, a DUE assignment was performed in TransCAD using the setting as reported in (*3*). Compared to results of Frank-Wolfe algorithm reported in (*3*), majority of the links have the same v_i and t_i values. Of the few links that have different v_i and t_i values, the maximum differences are 1 vph and 0.04 minutes, respectively. We attribute the small differences due to the algorithm's implementation details.
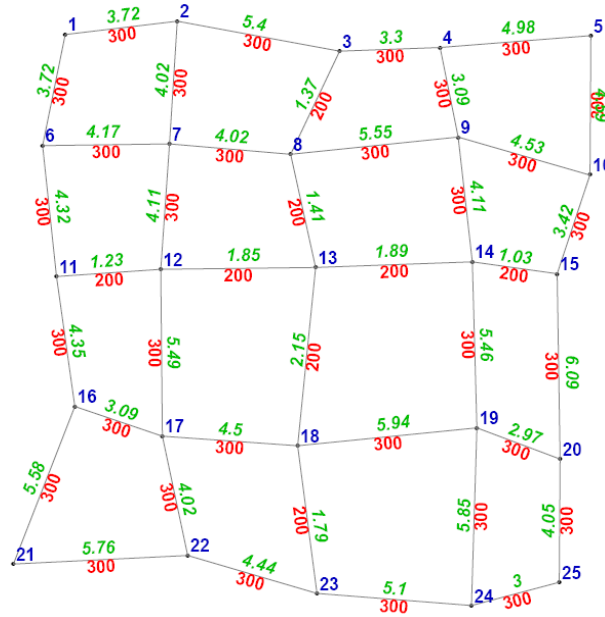
All links are two-way links.
Free-flow link travel time is shown above each link (in green, italic, in minutes)
Directional link capacity is shown below each link (in red, in vph)

Figure 1 Free-Flow Travel Time and Link Capacity of Test Network

TABLE 1 Origin-Destination Matrix of Text Network

Trips (vehicles per hour)		Destination Node			
		7	**9**	**17**	**19**
Origin Node	**7**	0	500	500	500
	9	500	0	500	500
	17	500	500	0	500
	19	500	500	500	0

The DN-DUE model was first implemented for this network. The standard values of α =0.15 and β =4 were used in the BPR function. To be consistent with the practice of the Texas Department of Transportation, the Frank-Wolfe algorithm was run for 100 iterations. Figure 2 shows the directional volume-capacity ratios (v_i/c_i) after 100 iterations. Since the O-D matrix is symmetrical and the links have the same t_i^f and c_i values in both directions, the resulting v_i/c_i and t_i are the same in both directions of a link. The t_i are displayed in Figure 3.
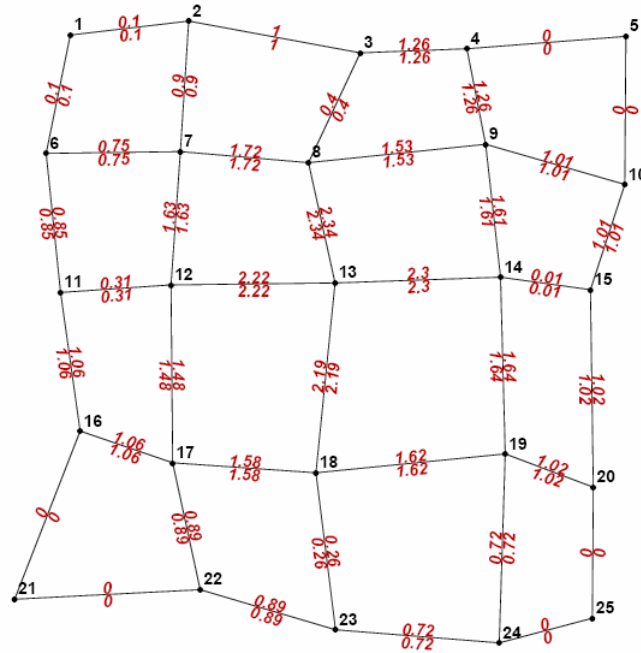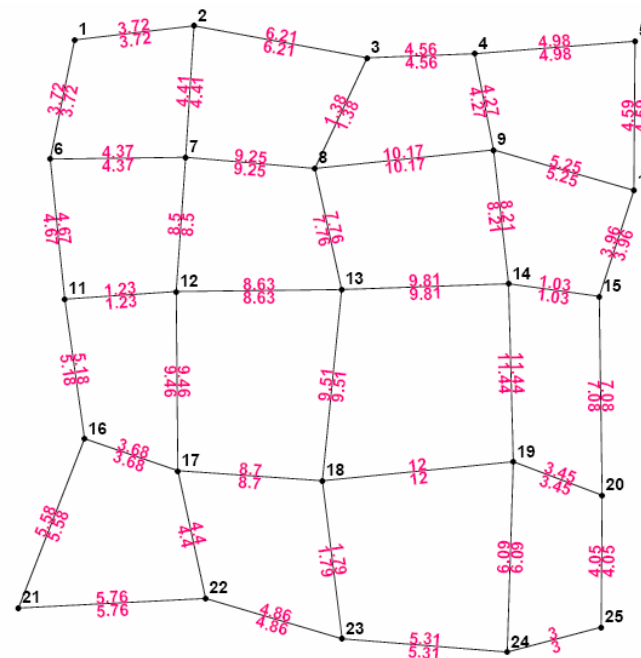
FIGURE 2 V-C Ratio after Traffic Assignment with BPR Function

Link travel time is in minutes

FIGURE 3 Link Travel Time after Traffic Assignment with BPR Function

In the SN-DUE model, we simply replaced the BPR function in the DN-DUE model by the simple ELD function described in (26). Put it simply, one only needs to change the value of

$\alpha =0.15$ in the BPR function to $a_1\alpha =1.4356 \times 0.15 = 0.2153$. Figures 4 and 5 show the directional v_i/c_i and average directional link travel time ($\bar{t}_i$), respectively, after 100 iterations.
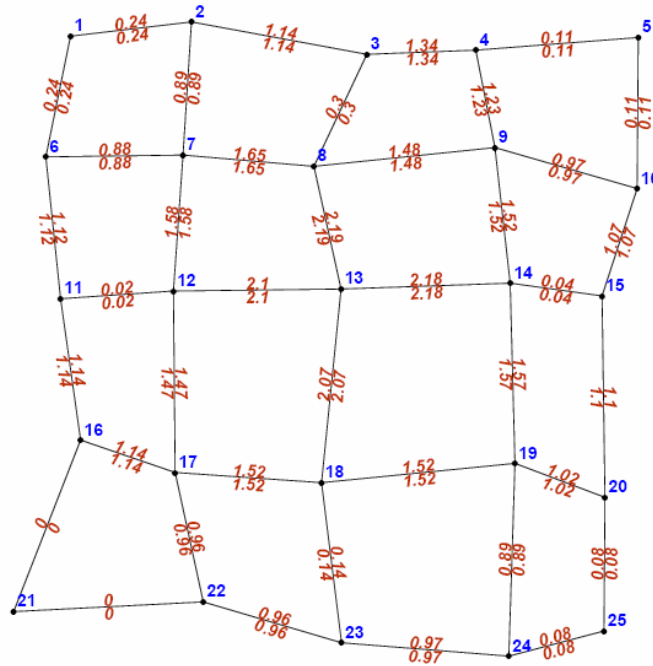

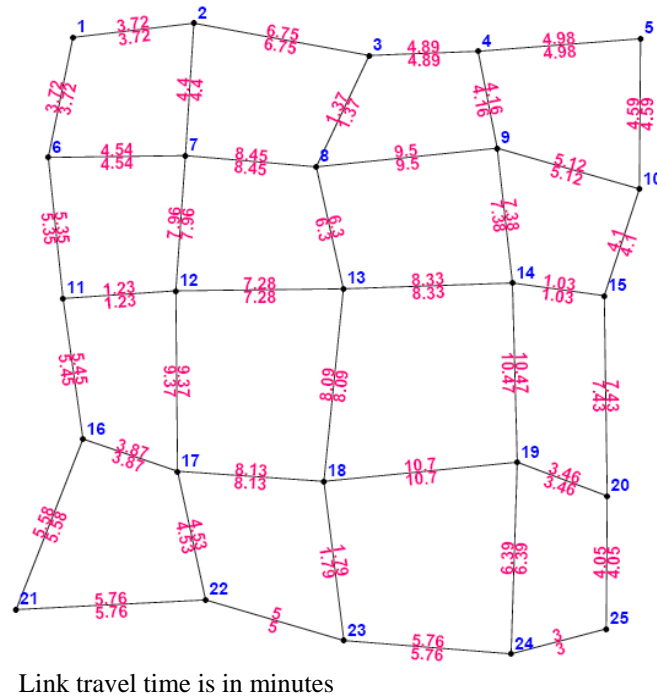**FIGURE 4 V-C Ratio after Traffic Assignment with ELD Function**

Link travel time is in minutes

FIGURE 5 Link Travel Time after Traffic Assignment with ELD Function

Comparison of Volume-Capacity Ratio

Figures 2 and 4 show the v_i/c_i of the links in the test network, after traffic assignments with the BPR and ELD functions, respectively. Compare to Figure 2, Figure 4 has 17 links with relatively lower v_i/c_i, 3 links with the same v_i/c_i ratio and 20 links with higher v_i/c_i. With the BPR function, there are 10 links with $v_i/c_i > 1.5$ in Figure 2. The v_i/c_i of these links have been reduced after the trips are assigned with the ELD function. For example, link 8-13 in Figure 2 has the maximum $v_i/c_i = 2.34$ in the network. In Figure 4, this link still has the maximum v_i/c_i in the network but the value has become 2.19. With the ELD function, risk averse drivers are more sensitive to v_i/c_i (the later is proportional to travel time variation) and therefore they will avoid links which have high volume, resulting in a more "uniform" distribution of traffic in the network.

Comparison of Link Travel Time

Figure 3 shows the t_i (for a DN-DUE model), computed from (1), while Figure 5 show the $\bar{t}_i$ (for a SN-DUE) computed from (2). For links that have high v_i/c_i in Figure 2, there are reductions from t_i in Figures 3 to $\bar{t}_i$ in Figure 5 (due to the fact the magnitude of change is proportional to v_i/c_i to the power of $\beta = 4$). Link with relatively low v_i/c_i in Figure 2 have no or marginal increase from t_i in Figures 3 to $\bar{t}_i$ in Figure 5. This is the overall effect of re-routing

some traffic from links with high volumes (and hence high travel time variance) to links with low volumes (with more certain travel times).

One point worth noting is that, in the DN-DUE model, the used routes between an O-D pair have the same route travel time that is less than the travel time of any unused route. However, in our SN-DUE model, all the used routes between an O-D pair have the same route disutility that is less than the disutility of any unused route. Therefore in the SN-DUE model, only the route disutility, not the route travel time, is in equilibrium. For risk averse drivers, the link disutility $\left(DU_i\right)$ is always greater than the average link travel time $\left(\bar{t}_i\right)$. Therefore, route disutility is always greater than the route travel time. To illustrate this, the link disutilities of the test network after traffic assignment with the ELD function is plotted in Figure 6. Readers can compare Figure 6 with Figure 5 to see that $DU_i \geq \bar{t}_i$, $\forall i$. Note that the difference between DU_i and $\bar{t}_i$ is greater when the v_i/c_i ratio is higher.
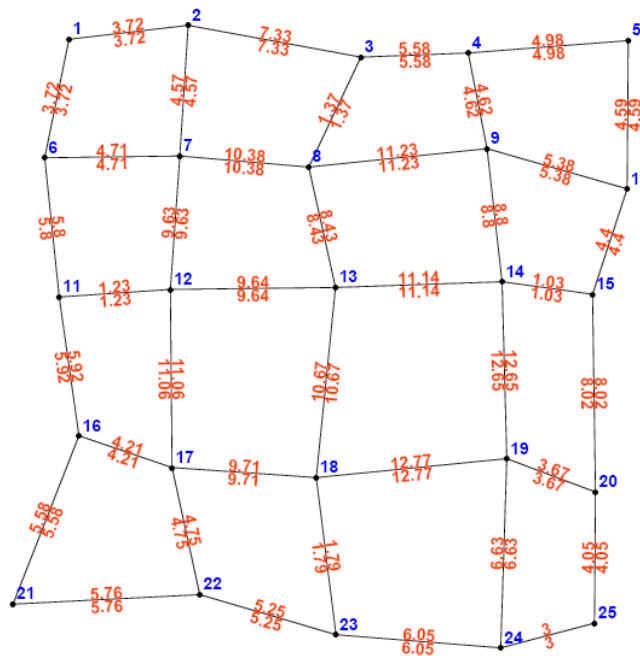

Link disutility is in minutes

FIGURE 6 Link Disutility after Traffic Assignment with ELD Function

Comparison of O-D Travel Time

Table 2 shows the O-D travel times along the shortest-time paths of the DN-DUE model, after 100 iterations of traffic assignment with the BPR function. Table 3 shows the O-D travel times along the shortest-disutility paths of the SN-DUE model, after 100 iterations of traffic assignment with the ELD function. As mentioned, in the SN-DUE model, drivers select the route between an O-D pair that has the smallest disutility. The routes with the smallest disutility may not be the same as the route with the shortest travel time. To illustrate this point, consider the route between nodes 7 and 17 in the test network. In Figure 6, the shortest-disutility path between nodes 7 and 17 is by nodes 7-6-11-16-17, with a route disutility of 20.64 minutes. In

Figure 5, this route has an average travel time of 19.21 minutes. However, if one examines Figure 5 carefully, the shortest-time path between nodes 7-17 is via nodes 7-12-17, with an average route travel time of 17.33 minutes, a saving of 1.88 minutes! In this case, along the route of nodes 7-12-17, links 7-12 and 12-17 have v_i/c_i of 1.58 and 1.47, respectively, which are higher than the v_i/c_i of the links along route of nodes 7-6-11-16-17 (see Figure 4). A higher v_i/c_i ratio indicates a higher link travel time variance (see (28)). This reflects that in a SN, a risk averse driver would rather select a route which has a higher average route travel time but smaller travel time variance over a route with a smaller average travel time but higher travel time variance.

TABLE 2 O-D Travel Time after Traffic Assignment with BPR Function

Travel Time (minutes)		**Destination Node**			
		7	**9**	**17**	**19**
Origin Node	**7**	-	19.42	17.90	38.26
	9	19.42	-	36.01	19.65
	17	17.90	36.01	-	20.65
	19	38.26	19.65	20.65	-

TABLE 3 O-D Travel Time after Traffic Assignment with ELD Function

Travel Time (minutes)		**Destination Node**			
		7	**9**	**17**	**19**
Origin Node	**7**	-	17.96	17.33	33.54
	9	17.96	-	31.93	17.84
	17	17.33	31.93	-	18.83
	19	33.54	17.84	18.83	-

The route travel times between the O-D pairs in Tables 2 and 3 are of interest in this comparison. For the O-D pairs between nodes 7-9 and 9-7, the shortest-disutility paths are not the same as the shortest-time paths. This has been discussed in the previous paragraph. Therefore, for these two O-D pairs, it is expected that the O-D travel times for the SN-DUE model in Table 3 are greater than those obtained with the DN-DUE model in Table 2. For each of the other O-D pairs, the shortest-disutility path is the same as the shortest-time path. Therefore, for these O-D pairs, the O-D travel times in Tables 3 are lower than those for the corresponding O-D pairs in Table 2. These lower O-D travel times is the result of drivers avoiding links with higher v_i/c_i and shift to links with lower v_i/c_i.

Comparison of Network Performance

The network performance is evaluated by comparing the total vehicle-miles traveled (VMT) and total vehicle-hours traveled (VHT) after 100 iterations of the Frank-Wolfe algorithm. For the DN-DUE model, the VMT is 32119 veh-miles and the VHT is 2545.08 veh-hrs. For the SN-

DUE model, the corresponding statistics are 32876 veh-miles and 2425.68 veh-hrs respectively. This reflects the fact that risk averse drivers prefer a longer route with a lower travel time variance than a shorter route with a higher travel time variance. The overall effect of redistribution of flow has resulted in a smaller VHT. The SN-DUE model has a total disutility of 2849.03 veh-hrs.

SUMMARY

This paper has derived a simple ELD function, which is of similar form as the BPR function, that represents the route choice behavior of risk averse drivers. The ELD function has a risk averse coefficient, but it does not depends on the link travel time distribution or variance. This ELD function permits transportation modelers to solve traffic assignment problem in a SN with the familiar DUE algorithms, simply by replacing the BPR function with the ELD function.

A method of calibrating the risk averse coefficient has been proposed and demonstrated with survey data gathered in El Paso, Texas.

The effect of using the ELD function in DUE assignment has been evaluated using a test network. Compare to the results of using the BPR function, the ELD function assigns more trips to low volume route thus results in a more uniform distribution of flow and lower congestion among the links in a network. This leads to lower route travel times for some O-D pairs and an overall reduction in VHT, but at the expense of a higher VMT.

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