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IDENTIFICATION OF HUMAN GAIT IN NEURO-REHABILITATION: TOWARDS EFFICIENT ALGORITHMS

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ABSTRACT

Many neurological diseases such as stroke, traumatic body injury, spinal cord injury drastically decrease the patient's ability to walk without physical assistance. To re-establish normal gait, patients undergo extensive rehabilitation. At present, rehabilitation requires gait assessment by highly qualified experienced clinicians. To make rehabilitations easier to access and to decrease the rehabilitation cost, it is desirable to automate gait assessment. In precise terms, gait assessment means comparing the recorded patient's gait with a standard (average) gait of healthy people of the same body measurements. One of the problems in this comparison is that patients walk slower; so, to properly compare gaits, we must first appropriately "scale" the standard gait so that it best matches the speed with which the patient's walk. One possibility is to try all possible scalings but this is computationally very intensive. In this paper, we adjust the known fast image referencing techniques to design a fast algorithm that uses Fast Fourier Transform for finding the optimal scaling.

Keywords*:* *Neuro-rehabilitation, Gait identification, Image referencing, Fast Fourier transform*

INTRODUCTION

Many neurological diseases such as stroke, traumatic body injury, and spinal cord injury drastically decrease the patient's ability to walk without physical assistance. To re-establish normal gait, patients undergo extensive rehabilitation. At present, rehabilitation requires gait assessment by highly qualified experienced clinicians. To make rehabilitations easier to access and to decrease the rehabilitation cost, it is desirable to automate gait assessment; see, e.g., [1].

In precise terms, a gait is measured by the dependence $x'(t)$ of some characteristic -- e.g., the acceleration or the angle between different parts of the foot. The gait assessment means comparing the recorded patient's gait with a standard (average) gait $x(t)$ of healthy people of the same age, body measurements, etc. One of the problems in this comparison is that patients walk slower; so, to properly compare gaits, we must first appropriately shift and "scale" the standard gait so that it best matches the speed with which the patient walks, i.e., to find the values t_0 and λ for which $x'(t) \approx x(\lambda \cdot t - t_0)$.

One possibility is to try all possible shifts and scalings but this is computationally very intensive. In this talk, we adjust the known image referencing techniques to design an efficient algorithm that uses Fast Fourier Transform for finding the optimal combination of a shift and a scaling.

IMAGE REGISTRATION ALGORITHMS: BRIEF OVERVIEW

There exist many methods for image registration; see, e.g., [2,3]. Among the most widely used methods are methods of *point matching*, where we find the matching points in the two images, and then the most appropriate transformation (rotation and/or shift) which maps the points from one image into the corresponding points from the other image.

Point matching methods work well when the images have clearly identifiable matching points, and when we know the images with a high accuracy -- so that we can identify and match these matching points with a reasonable accuracy. For example, in satellite images, we often have clear matching points representing special landmarks such as landmark city areas, landmark bridges, or tips of peninsulas. Such landmarks can usually be easily found in highly populated areas or in special terrains in which the area is highly non-homogeneous: e.g., there may be a clear shore line with a clear landmark point, or there may be a large clearly distinguishable river with a clear landmark turn.

However, there are many homogenous areas where it is not easy to find landmarks. For example, in the desert areas of the U.S. Southwest, the only visible landmarks are road intersections, and there are usually several similar-looking road intersections in the same image, so it is difficult to find the matching points between the two images. Similarly, in the mountain areas, there are many landmarks like summits and ridges, but usually, there are several similar-looking summits and ridges in each image, so it is difficult to match points in the two images.

For images known with very low accuracy, we may still find landmarks. However, since we only know the images with a very low accuracy, we may only be able to locate these landmarks with a very low accuracy, too low to enable us to adequately register the two images.

Sometimes, instead of landmark *points*, we have landmark *features*. For example, we may not have a landmark bridge, but we may have a clearly distinguishable river. In such situations, instead of matching points, we can match features. Such feature-matching algorithms are also efficiently used in image registration. However, in homogenous terrains and/or in situations when we only know the images with low accuracy, we may only be able to locate these features with a very low accuracy, too low to enable us to adequately register the two images.

In some cases, e.g., in many astronomical images, we have an image surrounded by an empty space. In this case, even when we cannot find the landmark points in the two images, we can match these images by comparing, e.g., the centers of gravity of these images. Alas, this is not the case in images like satellite images or radar images.

As an example of low accuracy images for which registration is practically important, we will actually consider low accuracy satellite images. So, in order to come up with an algorithm for registering low accuracy images, an algorithm which should be applicable for satellite images, we must place our emphasis on image registration techniques which go beyond point matching, feature matching, or simple geometric transformations in the image domain. Many such algorithms are based on the use of the Fast Fourier Transform (FFT); see, e.g., [4].

In this paper, we show how there FFT-based image processing algorithms can be modified to solve the gait assessment problem. Before we start describing these methods and adjusting them to the signal, let us provide a motivation for using FFT in such problems.

WHY FOURIER-BASED METHODS

Let us start with the simplest case of the problem. We have two functions $x(t)$ and $x'(t)$, and we must find the shift t_0 for which the signal $x'(t)$ will be, in some reasonable sense, the closest to the shifted signal $x(t - t_0)$. A reasonable way to describe the closeness between the two signals is to require that for every moment of time t, the corresponding intensities are close to each other. We can use, e.g., the squared difference $(x'(t) - x(t - t_0))^2$ between these values as the measure of the similarity at t, and we can use the sum (integral) $\int (x'(t) - x(t - t_0))^2 \, dt$ of these square differences over all moments t as the measure of overall similarity between the two signals.

The problem of finding the shift t_0 that minimizes the above integral takes the following form: find t_0 for which the integral $\int (x'(t) - x(t - t_0))^2 \, dt$ attains the smallest possible value. By representing the square of the difference $(x' - x)^2$ as the sum of three terms $(x')^2 + x^2 - 2x'x$, we can represent the above scoring function as

$$\int (x'(t))^2 \, dt + \int (x(t - t_0))^2 \, dt + 2\int x'(t)x(t - t_0) \, dt$$

The first integral in the sum does not depend on the shift at all. By using the new variable $s = t - t_0$, we can show that the second integral is equal to $\int (x(t - t_0))^2\, dt$ and thus, also does not depend on the shift. So, finding the shift for which the sum is the smallest possible is equivalent to finding the shift for which the *cross-correlation* term $\int x'(t)x(t- t_0)\, dt$ attains the largest possible value.

For signals described by values at n moments of time, a straightforward approach would require that we compute the value of the scoring function for all n possible shifts t_0. Computing each integral requires time $O(n)$, so overall, we need time $O(n)\cdot O(n) = O(n^2)$.

This computation can be performed much faster if we take into account that the cross-correlation term is a *convolution* between the signals $x(t)$ and $x'(t)$. Convolution is one of the main techniques in signal processing, and it is well known that we can compute convolution faster (in time $O(n\cdot log(n))$) by using Fast Fourier Transform (FFT). Specifically, to compute the convolution, we need the following steps:

1. first, we apply FFT to the original signals, resulting in functions $F(w)$ and $F'(w)$;
2. then, for each frequency w, we compute the product $P(w) = F'(w)\cdot F^*(w)$ (where F^* means complex conjugation);
3. finally, we apply the inverse Fourier transform to the resulting function $P(w)$, and get the desired cross-correlation function.

We can now find the shift as the value t_0 for which the cross-correlation attains the largest possible value.

The FFT of a signal of size n requires $O(n\cdot log(n))$ steps. Multiplication of the two Fourier transforms and the final search for the largest value both require processing each value w and t_0 once, so both require time $O(n)$. As a result, we can find the desired shift t_0 in time $O(n\cdot log(n)) + O(n) = O(n\cdot log(n))$.

SIMPLEST CASE: SHIFT DETECTION

Let us first consider the case when the two images differ only by shift: $x'(t) \approx x(t - t_0)$.

In the ideal case when $x'(t) = x(t - t_0)$, the Fourier transform $F'(w)$ of the signal $x'(t)$ can be obtained from the Fourier transform $F(w)$ of the signal $x(t)$ by the formula $F'(w) = exp(-2\pi iw\, t_0)\cdot F(w)$. In this case, the ratio $R(w) = P(w)/|P(w)|$ is equal to $exp(-2\pi iw\, t_0)$, and thus, its inverse Fourier transform is equal to the delta-function $\delta(t - t_0)$, i.e., equal to 0 everywhere except for the point t_0. In this case, the shift can be found as the only value at which this inverse Fourier transform is different from 0.

Since in reality, we only have approximate equality $x'(t) \approx x(t - t_0)$, the inverse Fourier transform will only approximately be equal to the delta-function. When the equality is accurate enough, the resulting inverse Fourier transform will still have close to 0 values for $t \neq t_0$. Thus, the desired shift t_0 value can be determined by the following algorithm [4]:

1. we apply FFT to the original signals $x(t)$ and $x'(t)$ and compute their Fourier transforms $F(w)$ and $F'(w)$;
2. we compute the product $P(w) = F^*(w) \cdot F'(w)$ and the ratio $R(w) = P(w)/|P(w)|$;
3. we apply the inverse FFT to the ratio $R(w)$ and compute its inverse Fourier transform $I(t)$;
4. we determine the desired shift t_0 as the point t_0 at which the magnitude $|I(-t_0)|$ attains the largest possible value.

This algorithm requires time $O(n\cdot log(n))$.

REDUCING SCALING TO SHIFT

Let us now consider a more complex problem, in which we must find a shift t_0 and scaling λ for which $x'(t) \approx x(\lambda\cdot t - t_0)$.

Since, in addition to shift, we also have scaling, the magnitudes $M(w)$ and $M'(w)$ of the corresponding Fourier transforms are not equal, but differ from each by the corresponding scaling: $M'(w) \approx (1/\lambda)\cdot M(w/\lambda)$. If we go to *log frequencies* $f = log(w)$ (for which $w = exp(f)$), then scaling becomes shift-like: $f \rightarrow f - b$, where $b = log(\lambda)$. So, in log frequencies, scaling is described by a shift.

In view of the above reduction, in order to determine the scaling between M and M', we can do the following:

- transform both images from the original frequencies to log frequencies;
- use the above FFT-based algorithm to determine the corresponding shift $log(\lambda)$;
- from the corresponding "shift" values, reconstruct the scaling coefficient λ.

Comment. The main computational problem with the transformation to log frequencies is that we need values $M(x)$ on a rectangular grid in log frequencies space, but computing $log(w)$ for the original grid points leads to points outside that grid. So, we need interpolation to find the values $M(x)$ on the desired grid. One possibility is to use linear interpolation.

FINAL ALGORITHM: DETERMINING SHIFT AND SCALING

1. we apply FFT to the original signals $x(t)$ and $x'(t)$ and compute their Fourier transforms $F(w)$ and $F'(w)$;
2. we compute the magnitudes $M(w) = |F(w)|$ and $M'(w) = |F'(w)|$ of these Fourier transforms;
3. we apply the above scaling detection algorithm to the functions $M(w)$ and $M'(w)$, and determine the scaling coefficient λ;
4. we apply the corresponding scaling to $x(t)$; as a result, we get a new signal $\underline{x}(t)$;
5. the signals $\underline{x}(t)$ and $x'(t)$ are already aligned in terms of scaling, the only difference between them is in an (unknown) shift; so, we again apply the above described FFT-based algorithm for determining shift: this time, actually to determine shift.

As a result, we get the desired values of shift and scaling. This algorithm also requires the time $O(n \cdot log(n))$.

Comment. Similar techniques have been applied to pavement engineering; see, e.g., [5,6].

CONCLUSIONS

Many neurological diseases such as stroke, traumatic body injury, spinal cord injury drastically decrease the patient's ability to walk without physical assistance. To re-establish normal gait, patients undergo extensive rehabilitation. At present, rehabilitation requires gait assessment by highly qualified experienced clinicians. To make rehabilitations easier to access and to decrease the rehabilitation cost, it is desirable to automate gait assessment. In this paper, we adjust the known fast image referencing techniques to design a fast algorithm that uses Fast Fourier Transform for gait assessment.

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