Current Financial Crisis and Inadequate Uncertainty Processing: A Comment

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The current financial crisis: a brief reminder. As is well known, one of the main causes of the 2008 financial crisis is the fact that banks underestimated the risk of investing in high-risk mortgages. According to the news reports, this underestimation was caused by the banks "mixing" mortgages into complex financial instruments, which somehow lowered the estimated risks. The mixing means, crudely speaking, that a bank made several financial instruments each containing a small portion of each original mortgage.

It is very clear that the mixing itself does not change the overall risk, since at the end, we still have the same high-risk mortgages. This means that, strictly speaking, it was not the mixing itself that caused the underestimation of risk but rather the way the risk was estimated for the financial instruments resulting from this mixture.

What we plan to do. Some news reports blame the un-intuitive complexity of the mixed financial instruments for the resulting mess. While some of these instruments are indeed complex, we believe that we can explain the problem in reasonably simple terms – and explain how this is related to uncertainty processing.

Traditional approach to financial engineering: a brief reminder. The main idea behind the traditional financial engineering is that to lower the risk, we must diversify: instead of investing into a single stock or a single industry, we invest parts of our investment amount into different stocks and/or industries, thus lowering the risk; see, e.g., [1, 2, 4].

In financial engineering, the risk is usually gauged as the standard deviation σ of the return on the corresponding investment (or, equivalently, as the variance – which is the square of the standard deviation). In financial engineering, it is usually assumed that the random risks related to different investments are independent. Let us show how this independence explains the need to diversify the investment portfolio.

Let us assume that we have several different shares; a unit share of the i-th company costs u_i and its return has a (predicted) standard deviation s_i. The risk is the same for all the shares of the i-th company, so the risk (standard deviation) doubles if we buy 2 shares, triples if we buy three shares, etc. In general, if we decide to invest an amount x_i into such shares, we buy $n_i \stackrel{\text{def}}{=} x_i/u_i$ shares. The risk in the resulting investment is thus equal to $\sigma_i = n_i \cdot s_i = (x_i/u_i) \cdot s_i = k_i \cdot x_i$, where we denoted $k_i \stackrel{\text{def}}{=} s_i/u_i$.

If we split the investment amount x between several stocks, by dividing x into $x = x_1 + \ldots + x_n$ and investing x_i in the i-th stock, the standard deviation related to the i-th stock is $\sigma_i = k_i \cdot x_i$. Since the risks associated with different stocks are assumed to be independent (and variances add for independent events), we conclude that the overall risk σ is equal to $\sigma = \sqrt{k_1^2 \cdot x_1^2 + \ldots + k_n^2 \cdot x_n^2}$. Formulas for describing and minimizing the resulting risk were the first result in financial engineering [3], results for which their author Harry M. Markowitz was awarded the Nobel prize in 1990.

A simple example can explain why the above diversification lowers the risk. Indeed, suppose that we have n different stocks with the same risk per cost $k_1 = \ldots = k_n = k$. If we invest the whole amount x into one of these stocks, the risk is equal to $k \cdot x$. If instead we divide this amount into n equal parts $x_1 = \ldots = x_n = x/n$ and invest each part into the corresponding stock, then the risk of resulting investment will be equal to

$$\sigma = \sqrt{\sum_{i=1}^{n} k_i^2 \cdot x_i^2} = \sqrt{\sum_{i=1}^{n} k^2 \cdot \left(\frac{x}{n}\right)^2} = \sqrt{n \cdot k^2 \cdot \frac{x^2}{n^2}} = \sqrt{\frac{k^2 \cdot x^2}{n}} = \frac{k \cdot x}{\sqrt{n}}.$$

Thus, a simple division of the stock into n parts decreases the risk by a factor of $\sqrt{n}$: if we divide into $n = 4$ parts, we decrease the risk to a half, if we divide into $n = 100$ parts, we decrease the risk by a factor of 10, etc.

How the traditional approach has lead to the current crisis. From this viewpoint, if we want to invest into risky mortgages, it makes sense not to invest into a single one, but rather invest a small amount of money into each of these mortgages, thus lowering the resulting risk. So far, so good, and this "slicing" indeed lowers the risk. However, the problem is that this is exactly what all the banks and mortgage companies were doing: each of the banks and companies, to minimize its risks, invested the money into similar "sliced" (mixed) combinations of the same mortgages. Thus, we get a lot of mixed financial instruments. Now comes the important part.

An investment fund wants to invest its money. It can just invest into a single financial instrument, but that will not be diversified enough. So, following the reasonable advice of financial engineering, the investment fund invests in several different financial instruments. According to the traditional approach to financial engineering, we assume that the risks of investing in different instruments are independent. Under this assumption, as we have shown, when we start with the risk per dollar σ for each individual instrument, then, by spreading the investment between n instruments, we should lower the risk to $\sigma_{\text{estimated}} = \sigma/\sqrt{n}$. If we spread our investment amount between a large number of instruments $n \gg 1$, we can drastically decrease the risk.

However, in this particular case, the independence assumption is clearly false: all the instruments combine the same mortgages, and therefore, their risks are actually strongly correlated. Since they invest in the same mortgages, the risk per dollar is the same actual risk $\sigma_{\text{actual}} = \sigma$, no matter how many instruments we combine.

So, we face the situation in which the traditional financial engineering tools drastically underestimate the risk:

$$\sigma_{\text{estimated}} = \frac{\sigma}{\sqrt{n}} \ll \sigma_{\text{actual}} = \sigma.$$

How uncertainty processing can help to avoid such crises. This explanation shows that a more adequate uncertainty processing can help us avoid such situations in the future:

- First, in addition to purely formal mathematical tools, we should use common sense, expert opinion, etc.; some of this expert opinion is imprecise, so we may need to use fuzzy techniques or other techniques for handling such imprecise statements.

- Second, in situations when we do not have information about the possible dependence of different financial instruments, instead of assuming independence (as it is done now), we should use imprecise probability tools to take into account the possibility of dependence.

Final comment: this is especially important for financial engineering. An important question is: the independence assumption is typical in science and engineering; see, e.g., [5]. It is well understood that this assumption is often false. Nevertheless, it is frequently used and rarely leads to catastrophic consequences: why?

The reason is that in science and engineering, we start with a data processing algorithm – that was usually developed without taking uncertainty into account – and we estimate the resulting uncertainty. Even if we have a model which in some rare cases drastically underestimates uncertainty, it is a rare occasion to hit these cases. In financial engineering, instead, we are actively soliciting the situations in which the (estimated) uncertainty is the smallest. So, if for the estimated uncertainty there is a case when this estimated uncertainty is very small, our minimization process will lead to exactly this case.

Thus, in financial engineering, it is even more important to take into account dependence and expert information than in science and engineering.

References

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