

Decision making under interval uncertainty

Vladik Kreinovich

Department of Computer Science
University of Texas at El Paso
El Paso, TX 79968, USA
vladik@utep.edu

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To make a decision, we must:

- find out the user's preference, and
- help the user select an alternative which is the best – according to these preferences.

A general way to describe user preferences is via the notion of *utility* (see, e.g., [7]): we select a very bad alternative A_0 and a very good alternative A_1 ; utility $u(A)$ of an alternative A is then defined as the probability p for which A is equivalent to the lottery in which we get A_1 with probability p , and A_0 otherwise. One can prove that utility is determined uniquely modulo linear re-scaling (corresponding to different choices of A_0 and A_1), and that the utility of a decision with probabilistic consequences is equal to the expected utility of these consequences.

Once the utility function $u(d)$ is elicited, we select the decision d_{opt} with the largest utility $u(d)$. Interval techniques can help in finding the optimizing decision; see, e.g., [4].

Often, we do not know the exact probability distribution, so we need to extract, from the sample, the characteristics of a distribution which are most appropriate for decision making. We show that, under reasonable assumptions, we should select moments and cumulative distribution function (cdf). Based on a finite sample, we can only find bounds on these characteristics, so we need to deal with bounds (intervals) on moments [6] and bounds on cdf [1] (a.k.a. p-boxes).

Once we know intervals $[u(d), \bar{u}(d)]$ of possible values of utility, which decision shall we select? We can simply select a decision d_0 which *may* be optimal,

i.e., for which $\bar{u}(d_0) \geq \max_d \underline{u}(d)$, but there are usually many such decisions; which of them should be select? It is reasonable to assume that this selection should not depend on linear re-scaling of utility; under this assumption, we get Hurwicz optimism-pessimism criterion $\alpha \cdot \bar{u}(d) + (1 - \alpha) \cdot \underline{u}(d) \rightarrow \max$ [7]. The next question is how to select α : interestingly, e.g., too optimistic values ($\alpha > 0.5$) do not lead to good decisions.

In some situations, it is difficult to elicit even interval-valued utilities. In many such situations, there are reasonable symmetries which can be used to make a decision; see, e.g., [5]. We show how this method works on the example of selecting a location for a meteorological tower [3].

Finally, while optimization problems are ubiquitous, sometimes, we need to go beyond optimization: e.g., we need to make sure that the system is *controllable* for all disturbances within a given range. In such problems, modal intervals [2] naturally appear. In more complex situations, we need to go beyond modal intervals, to more general Shary's classes.

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