Comparing intervals and moments for the quantification of coarse information

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ABSTRACT

Practical engineering problems often involve the quantification of uncertainty on the basis of quite limited information. This is associated with two problems. First, it is desirable to represent the available information as completely as possible in the theoretical uncertainty model. Second, the modeling should not rely on assumptions, which cannot be justified and so introduce artificial information. These issues need to be addressed properly in order to arrive at realistic results in a structural, reliability or other analysis. In practice, a complete satisfaction of both requirements is virtually impossible, but a good balance can be obtained in many cases.

If the available information is very limited, the specification of probability distributions is critical, but coarse models may still be suitable. In the probabilistic framework, one may then estimate the first two moments to work towards a rough probabilistic approximation solution. Alternatively, one could estimate two bounds to form the range of the corresponding variable and thus, generate an interval. In this paper we investigate these alternatives in view of their information content in order to provide some advice which alternative to choose in which situation. A natural idea is to select the most informative approach, i.e., that approach in which we need the smallest amount of additional information (in Shannon's sense) to obtain the full information about the situation. We follow this idea and come up with the following conclusion: in practical situations in which a 95% confidence level is sufficient, interval bounds are more informative; however, in situations in which we need higher confidence, the moments approach is more informative.

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