Towards Designing Optimal Individualized Placement Tests

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Computers enable us to provide individualized learning, at a pace tailored to each student. In order to start the learning process, it is important to find out the current level of the student's knowledge, i.e., to place the student at an appropriate level. Usually, such placement tests use a sequence of $N$ problems of increasing complexity; if a student is able to solve a problem, the system generates a more complex one; if a student cannot solve a problem, the system generates an easier one -- until we find the exact level of this student. After this, the actual learning starts.

A natural tendency is to speed up this preliminary stage and to get to actual leaning ASAP, i.e., to minimize the number of problems given to a student. The solution to the corresponding optimization problem is a well-known bisection procedure: if we know that a student can solve a problem on level $i$ and cannot solve a problem on level $j$, we give the student a problem on level $(i + j)/2$. The problem with bisection is that every time a student is unable to solve a problem, he/she gets discouraged; in other words, such problems have a larger effect on the student than problems which the student can solve. To take this into account, we define an overall student effect by combining "positive" and "negative" problems with different weights, and we develop a testing algorithm which minimizes this effect. The resulting algorithm is similar to bisection, except that now we select a problem on level $c \cdot i + (1 - c) \cdot j$ for an appropriate parameter $c$ depending on the degree of discouragement: the more discouraged a student, the smaller $c$. 