

# Constraint Programming and Decision Making

*Constraint programming and decision making are important.* Constraint programming and decision-making techniques are essential in the building of intelligent systems. They constitute an efficient approach to representing and solving many practical problems. They have been applied successfully to a number of fields, such as scheduling of air traffic, software engineering, networks security, chemistry, and biology. However, despite the proved usefulness of these techniques, they are still under-utilized in real-life applications. One reason is the perceived lack of effective communication between constraint programming experts and domain practitioners about constraints, in general, and their use in decision making, in particular.

*CoProd workshops.* To bridge this gap, annual International Constraint Programming and Decision Making workshops CoProd'XX have been organized since 2008: in El Paso, Texas (2008, 2009, 2011, and 2013), in Lyon, France (2010), and in Novosibirsk, Russia (2012); CoProd'2014 will be held in Würzburg, Germany. This volume contains extended version of selected papers presented at previous CoProd workshops.

CoProD workshops aim to bring together, from areas closely related to decision making, researchers who design solutions to decision-making problems and researchers who need these solutions and likely already use some solutions. Both communities are often not connected enough to allow cross-fertilization of ideas and practical applications.

CoProD workshops aim at facilitating networking opportunities and cross-fertilization of ideas between the approaches used in the different attending communities. Because of this, in addition to active researchers in decision making and constraint programming techniques, these workshops are also attended by domain scientists – whose participation and input is highly valued in these workshops.

*The goal of CoProD workshops is therefore to constitute a forum for inter-community building.* The objectives of this forum are to facilitate:

- The presentation of advances in constraint solving, optimization, decision making, and related topics;
- The development of a network of researchers interested in constraint techniques, in particular researchers and practitioners that use numeric and symbolic approaches (or a combination of them) to solve constraint and optimization problems;
- The gap bridging between the great capacity of the latest decision-making/constraint techniques and their limited use.

CoProD workshops can impact these communities by easing collaborations and therefore the emergence of new techniques, and by creating a network of interest.

The objectives of CoProD are also relayed all year round through the website [constraintsolving.com](http://constraintsolving.com).

*Topics of interest.* The main emphasis is on the joint application of constraint programming and decision making techniques to real-life problems. Other topics of interest include:

- Algorithms and applications of:
  - Constraint solving, including symbolic-numeric algorithms
  - Optimization, especially optimization under constraints (including multi-objective optimization)
  - Interval techniques in optimization and their interrelation with constraint techniques
  - Soft constraints
  - Decision making techniques
- Description of domain applications that:
  - Require new decision making and/or constraint techniques
  - Implement decision making and/or constraint techniques

*Contents of the present volume: general overview.* All these topics are represented in the papers forming the current volume. These papers cover all the stages of decision making under constraints:

- how to formulate the problem of decision making in precise terms, taking different criteria into account;
- how to check whether (and when) the corresponding decision problem is algorithmically solvable;
- once we know that the decision problem is, in principle, algorithmically solvable, how to find the corresponding algorithm, and how to make this algorithm as efficient as possible;
- how to take into account uncertainty, whether it is given in terms of bounds (intervals), probabilities, or fuzzy sets?

*How to formulate the problem of decision making: general case.* The paper [2] emphasizes that in decision making, it is important *not to oversimplify* the problem: a model which is a reasonably good (but not very accurate) fit for all previous observations can lead to misleading decisions. A similar conclusion is made in [11]: if we try to simply our problem by ignoring some of the natural constraints, often, the problem, instead of becoming algorithmically easier, becomes more complex to solve. So, using a more realistic model not only makes the results of the computations more adequate, it also often makes computations themselves easier (and faster).

*Case of multi-criterion decision making.* How can we combine different criteria?

- It is desirable to find a combination rule which is in best accordance with the actual decisions; a new method for solving the corresponding optimization problems is presented in [14].
- In many practical situations, it is possible to use known *symmetries* to find the most appropriate combination.
  - The paper [10] uses symmetries to explain why tensors and polynomial combination rules are often practically useful.
  - The paper [7] applies similar symmetry ideas to a specific problem of selecting the best location for a meteorological tower.

Instead of combining *criteria* and solving the resulting combined optimization problem, we can alternatively solve the optimization problems corresponding to all possible combinations, and then *select the solution* which is, in some reasonable sense, the most appropriate. As shown in [21], in this case also natural symmetries explain the efficiency of empirically successful selection heuristics.

*When are problems algorithmically solvable?*

- For *general* decision making problems, this question is analyzed in [1]; this paper also analyzes when it is possible to solve the problem while avoiding making irreversible changes.
- In some cases, when no algorithm is possible for a general *mathematical problem*, algorithms becomes possible is we only consider *physically meaningful* cases, i.e., if we take into account additional physical constraints [12].
- In some cases, partial solutions can be extended to general ones:
  - Paper [8] shows that, in principle, it is sufficient to be able to algorithmically compute the *quality* of the best decision, then it is possible to algorithmically find this optimal decision.
  - The paper [3] shows that it is always possible to *combine* algorithms for different possible situations into a single algorithm – even when it is not always algorithmically possible to decide which of the possible situations we currently encounter.

*How to design efficient algorithms for solving the problems.* There are several ways to design more efficient algorithms.

- First, it is often beneficial to *reformulate* the original problem.
  - In [13], it is shown that often real-world problems become easier to solve if we reformulate them in terms of constraints – e.g., in terms of constraint optimization – and then use constraint techniques to solve these problems.
  - Moreover, it turns out that sometimes, adding *additional* constraints [11] make problems easier to solve – constraints which, at first glance, would make the problem more complex to solve.

- Once the problem is formulated, we can try to come up with *more efficient algorithms* for solving the problem. This can be done both on a higher level – by coming up with a better numerical algorithm, or on lower level – by making elementary steps of the corresponding numerical algorithm more efficient. This volume contains examples of both approaches:
  - Innovative efficient algorithms for constraint optimization and equation solving are presented in [6, 19].
  - Efficient algorithms for dealing with matrix and, more generally, tensor data are presented in [15].
- Often, while we do not have efficient general algorithms for solving a practical problem, human experts efficiently solve this problem. In such situations, it is important to learn how humans make decisions.
  - For multi-agent decision making in multi-criteria situations, such an analysis is presented in [5].
  - Similar studies of human decision making are also important in situations when we need to *influence* collective human decisions – e.g., evacuation in emergency situations [20].

*How to take uncertainty into account.* For a single variable, the simplest type of uncertainty is when we have bounds on this variables, i.e., when possible values of this variable form an *interval*. For interval uncertainty,

- a new more efficient method is described in [19]; this method is useful in solving systems of equations and in solving optimization problems under interval uncertainty;
- a new control techniques under interval uncertainty is described in [16].

In multi-dimensional case, in addition to intervals restricting the values of each variable, we may have additional constraints which limit the range of possible values of the corresponding tuples. Ellipsoids are often a computationally efficient tool for describing the resulting tools. The paper [23] provides a theoretical explanation for this empirical success. In [17], it is shown that sometimes half-ellipsoids provide an even more computationally efficient description of uncertainty.

Several papers take into account *probabilistic* uncertainty. The paper [18] uses interval techniques to simulate non-standard probability distribution useful in biological applications. The paper [9] shows that constraints techniques, when applied to statistical situations, explain well-known techniques of computational statistics such as Gibbs sampling. Finally, the paper [22] describes a new approach to solving problems with probabilistic uncertainty in which, in addition to continuous variables, more difficult-to-process discrete variables also need to be taken into account.

Constraint optimization problems under interval-valued *fuzzy* uncertainty are discussed in [4].

*Resulting applications.* Papers presented in this volume includes numerous applications. We want to emphasize three such applications:

- to meteorology and environmental science [7] (selecting the best location for a meteorological tower),
- to biology [18]: how to find the most probable evolution history of different species, and
- to engineering [16]: how to best control a magnetic levitation train.

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