Towards the possibility of objective interval uncertainty in physics. II

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Applications of interval computations usually assume that while we only know an interval $[\underline{x}, \overline{x}]$ containing the actual (unknown) value of a physical quantity x, there *is* the exact value x of this quantity – and that in principle, we can get more and more accurate estimates of this value. This assumption is in line with the usual formulations of physical theories – as partial differential equations relating exact values of different physical quantities, fields, etc., at different spatial locations and moments of time; see, e.g., [2]. Physicists know, however, that due, e.g., to Heisenberg's uncertainty principle, there are fundamental limitations on how accurately we can determine the values of physical quantities [2, 5].

One of the important principles of modern physics is *operationalism* – that a physical theory should only use observable quantities. This principle is behind most successes of the 20 century physics, starting with relativity theory (vs. un-observable aether) and quantum mechanics. From this viewpoint, it is desirable to avoid using un-measurable exact values and modify physical theories so that they explicitly take objective uncertainty into account.

According to quantum physics, we can only predict probabilities of different events. Thus, uncertainty means that instead of exact values of these probabilities, we can only determine intervals; see, e.g., [3].

From the observational viewpoint, a probability measure means that we observe a sequence which is random (in Kolmogorov-Martin-Löf (KML) sense) relative to this measure. What we thus need is the ability to describe a sequence which is random relative to a *set* of possible probability measures. This is not easy: in [1, 4], we have shown that in seemingly reasonable formalizations, every random sequence is actually random relative to one of the original measures. Now we know how to overcome this problem: for example, for a sequence of events $\omega_1\omega_2\ldots$ occurring with the interval probability $[\underline{p},\overline{p}]$, we require that this sequence is random relative to a product measure corresponding to some sequence of values $p_i \in [\underline{p},\overline{p}]$ – and that it is not random in this sense for any narrower interval. We show that this can be achieved when $\liminf p_i = \underline{p}$ and $\limsup p_i = \overline{p}$.

We also analyze what will happen if we take into account that in physics, not only events with probability 0 are physically impossible (this is the basis of KML definition), but also events with very small probability are impossible (e.g., it is not possible that all gas molecules would concentrate, by themselves, in one side of a vessel).

References:

[**1**] D. Cheu, L. Longpré, Towards the possibility of objective interval uncertainty in physics, *Reliable Computing*, 15(1) (2011), pp. 43–49.

[**2**] R. Feynman, R. Leighton, M. Sands, *Feynman Lectures on Physics*, Basic Books, New York, 2005.

[**3**] I.I. Gorban, *Theory of Hyper-Random Phenomena*, Ukrainian National Academy of Sciences Publ., Kyiv, 2007 (in Russian).

[**4**] V. Kreinovich, L. Longpré, Pure quantum states are fundamental, mixtures (composite states) are mathematical constructions: an argument using algorithmic information theory, *International Journal on Theoretical Physics*, 36(1) (1997) pp. 167–176.

[**5**] L. Longpré, V. Kreinovich, When are two wave functions distinguishable: a new answer to Pauli's question, with potential application to quantum cosmology, *International Journal of Theoretical Physics*, 47(3) (2008), pp. 814–831.