How much for an interval? a set? a twin set? a p-box? a Kaucher interval? An economics-motivated approach to decision making under uncertainty

Joe Lorkowski and Vladik Kreinovich

lorkowski@computer.org, vladik@utep.edu

Keywords: decision making, interval uncertainty, set uncertainty, p-boxes

There are two main reasons why decision making is difficult. First, we need to take into account many different factors, there is usually a trade-off. For example, shall we stay in a slightly better hotel or in a reasonably good cheaper one?

But even when we know how to combine different factors into a single objective function, decision making is still difficult because of uncertainty. For example, when deciding on the best way to invest money, the problem is that we are not certain which financial instrument will lead to higher returns.

Let us use economic ideas to solve such economic problems: namely, let us assign a fair price to each case of uncertainty.

What does "fair price" mean? One of the reasonable properties is that if v is a pair price for an instrument x and v' is a fair price for an instrument x', then the fair price for a combination $x + x'$ of these two instruments should be equal to the sum of the prices.

In [3], this idea was applied to *interval* uncertainty [4], for which this requirement takes the form $v([\underline{x}, \overline{x}]+[\underline{x}', \overline{x}']) = v([\underline{x}, \overline{x}])+v([\underline{x}', \overline{x}'])$. Under reasonable monotonicity conditions, all such functions have the form $v([\underline{x}, \overline{x}]) = \alpha\cdot\overline{x}+(1-\alpha)\cdot\underline{x}$ for some $\alpha \in [0, 1]$; this is a well-known Hurwicz criterion.

In this talk, we show that for *sets* S, we similarly get $v(S) = \alpha \cdot \sup S + (1 - \alpha) \cdot \inf S$.

For *probabilistic* uncertainty, for large N, buying N copies of this random instrument is equivalent to buying a sample of N values coming from the corresponding probability distribution. One can show that for this type of uncertainty, additivity implies that the fair price should be equal to the expected value μ.

A similar idea can be applied to finding the price of a *p-box* (see, e.g., [1, 2]), a situation when, for each x, we only know an interval $[\underline{F}(x), \overline{F}(x)]$ containing the actual (unknown) value

$$F(x) = \text{Prob}(\eta \le x)$$

of the cumulative distribution function. In this case, additivity leads to the fair price $\alpha \cdot \overline{\mu} + (1-\alpha) \cdot \underline{\mu}$, where $[\underline{\mu}, \overline{\mu}]$ is the range of possible values of the mean μ.

We also come up with formulas describing fair price of *twins* (intervals whose bounds are only known with interval uncertainty) and of *Kaucher* (improper) *intervals* $[\underline{x}, \overline{x}]$ for which $\underline{x} > \overline{x}$.

References:

[1] S. Ferson, *Risk Assessment with Uncertainty Numbers: RiskCalc*, CRC Press, Boca Raton, Florida, 2002.

[2] S. Ferson et al., *Experimental Uncertainty Estimation and Statistics for Data Having Interval Uncertainty*, Sandia National Laboratories, Report SAND2007-0939, May 2007; available as http://www.ramas.com/intstats.pdf

[3] J. McKee, J. Lorkowski, T. Ngamsantivong, Note on Fair Price under Interval Uncertainty, *Journal of Uncertain Systems*, (8) 2014, to appear.

[4] R.E. Moore, R.B. Kearfott, M.J. Cloud, *Introduction to Interval Analysis*, SIAM, Philadelphia, 2009.