Fitts's Law: Towards a Geometric Explanation

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Abstract

In designing human-computer interfaces, designers use an empirical Fitts's Law, according to which the average time T of accessing an icon of size w at a distance d from the center of the screen is proportional to the logarithm of the ratio w/d. There exist explanations for this law, but these explanations have gaps. In this paper, we show that these gaps can be explained if we analyze this problem from the geometric viewpoint. Thus, we get a geometric explanation of the Fitts's Law.

What is Fitts's Law. The efficiency of computer-based systems for education, information, commerce, etc., strongly depends on the user-friendliness of the corresponding interfaces, in particular, on the location and size of the appropriate icones. When deciding the location and size of different icons on a computer screen, designers use the Fitts's Law [3, 4]. This law describe how the average time T of accessing an icon depends on the distance d from the center of the screen to the icon and on the linear size w of this icon: $T = a + b \cdot \ln\left(\frac{d}{w}\right)$, for some constants a and b.

How Fitts's Law is used in interface design. The use of Fitts's Law started with the very first mouse-accessible interfaces; see, e.g., [1]. It is based on the following idea.

Each icon corresponds to a specific task or group of tasks. Some tasks are more frequent, some are rarer: for example, editing is a frequent task, while logging off is a rarer task. For each task, we can empirically determine the frequency f_i with which this task is performed. We can therefore gauge the user-friendliness of the interface by the average time

$$\sum_i f_i \cdot T_i = \sum_i f_i \cdot \left(a + b \cdot \ln\left(\frac{d_i}{w_i}\right)\right)$$

needed to access the required icon. Out of several possible interfaces, we select the one for which this average time is the smallest.

Fitts's Law: qualitative aspects. From the qualitative viewpoint, the Fitts's Law says that T decreases when d decreases and/or w increases. In other words:

- the closer the icon to the center, the easier it is to find this icon, and
- the larger the icon size, the easier it is to find it.

From this viewpoint, Fitts's Law is simply common sense.

Quantitative aspects of the Fitts's Law need explanation. That the time T should monotonically depend on the distance d and on the size w is clear, but there are many different monotonic functions. The fact that overwhelming majority of experimentally results is in very good accordance with one type of monotonic dependence – the logarithmic law – needs explanation.

Current explanation of Fitts's Law. A current explanation of Fitts's Law [2] is is based on the fact that our motions are not perfect. For simplicity, this explanation assumes that each movement aiming at reaching an object at distance d actually only follows a slightly smaller distance $(1-\varepsilon)\cdot d$, for some accuracy $\varepsilon < 1$. Thus, after the original movement, we are still a distance $(\varepsilon \cdot d$ away from the desired object. We therefore need the next movement to reach this object.

This second movement brings us to the distance $\varepsilon \cdot (\varepsilon \cdot d) = \varepsilon^2 \cdot d$ to the target. In general, after k movements, we are at a distance $\varepsilon^k \cdot d$ from the target. If we aim at the center of an icon, then we reach a point within the icon when this distance is smaller than or equal to the icon's half-size $\frac{w}{2}$, i.e., when $\varepsilon^k \cdot d \le \frac{w}{2}$. From the condition that $\varepsilon^k \cdot d \approx \frac{w}{2}$, we can determine the number of iterations k as $k \approx \frac{1}{\ln(\varepsilon)} \cdot \ln\left(\frac{2d}{w}\right)$. One can easily check that we thus get $k \approx a + b \cdot \ln\left(\frac{d}{w}\right)$, where $a = \frac{\ln(2)}{\ln(\varepsilon)}$ and $b = \frac{1}{|\ln(\varepsilon)|}$.

The overall time needed to reach the icon consists of the time of the smooth motions and the time needed to switch from one motion to another. Usually, the switch time is much larger. So, in first approximation, we can simply ignore the time of the smooth movements and conclude that the time T is proportional to the number of switches k. Thus, we arrive at the Fitts's formula.

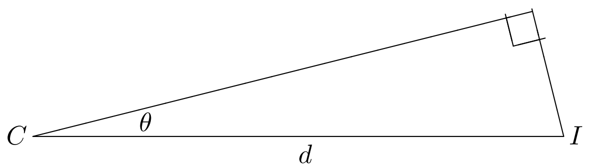
This explanation has some gaps. As noted, e.g., in [6], the above explanation is not perfect, it has two gaps. The first gap is not critical: the above derivation assumes that for the same distance d, the motor error is always the same, while in reality, in repeated experiments, we may get different values of the motor error. This gap is not critical, because the above derivation will not change is we take into account that after the first iteration, the distance to the target is only *approximately* equal to $\varepsilon \cdot d$.

The second gap is more serious. The above derivation is based on the assumption that if we want to move to a distance d, then the accuracy with which we can perform this movement is equal to $\varepsilon \cdot d$. In other words, this derivation is based on the assumption that the *relative* accuracy $\frac{\varepsilon \cdot d}{d}$ is the same for all the distances. If the relative accuracy depends on the distance d, i.e., if the accuracy is equal to $\varepsilon(d) \cdot d$ for some function $\varepsilon(d) \neq \text{const}$, then, instead of the Fitts's Law, we would get a different formula.

What we do. To come up with a more convincing explanation of the Fitts's Law, we therefore need to explain why the relative accuracy does not depend on the distance. This is what we do in this paper.

Our explanation of Fitts's Law. Let us assume that the cursor (controlled, e.g., by a mouse) is currently located at the center C of the screen, and we want to move it to the location of the icon I. The shortest way from one point to another is a straight line, so naturally, we start a straight line in the direction of the icon. To be more precise, we select an angle leading us to the icon, and we follow a straight line in the direction of this angle.

If we could set up the angle exactly, we would then follow the straight line to the desired icon and reach this icon in one movement. In practice, of course, there is a motor error; we cannot set the angle of our movement exactly, we can only set up this angle with some accuracy θ. Because of this accuracy, the straight line that we actually follow is at an angle of order θ from the line connecting the center of the screen with the target icon.



As a result of this motion inaccuracy, we do not reach the desired point I, the closest we get to I is at a distance $\approx d \cdot \sin(\theta)$. As a result of a movement, we get from the location at a distance d from the target point I to a new location whose distance to I is approximately equal to $\varepsilon \cdot d$, where $\varepsilon \stackrel{\text{def}}{=} \sin(\theta)$.

To reach the desired location I, starting from this new point, we again aim at I. As a result, we get from the point at a distance $\approx \varepsilon \cdot d$ to I to a new point whose distance from I is approximately equal to $\varepsilon \cdot (\varepsilon \cdot d) = \varepsilon^2 \cdot d$. After

k iterations, we reach a point at a distance $\approx \varepsilon^k \cdot d$ to the target point I. We reach the icon if this distance does not exceed the icon's half-width $\frac{w}{2}$, i.e., when $\varepsilon^k \cdot d \approx \frac{w}{2}$.

As we have mentioned, the resulting number of iterations is $k \approx a+b\cdot\ln\left(\frac{w}{d}\right)$. Under a natural assumption that the average time T needed to reach an icon is proportional to this number of iterations, we get the desired Fitts's Law.

Comment. It is worth mentioning that a similar geometric argument describes how the number of corrections needed for inter-stellar travel depends on the travel distance d; see, e.g., [5].

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