Interleaving Enhances Learning: A Possible Geometric Explanation

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Abstract

In the traditional approach to learning, if we want students to learn how to solve different types of problems, we first teach them how to solve problems of the first type, then how to solve problems of the second type, etc. It turns out that we can speed up learning if we interleave problems of different types. In particular, it has bene empirically shown that interleaving problems of four different types leads to a double speed-up. In this paper, we provide a possible geometric explanation for this empirical fact.

1 Interleaving Enhances Learning: An Empirical Fact

Traditional approach to learning several skills. Traditionally, when students need to learn several skills, they learn them one by one:

- first, they learn the skill a;
- once they have mastered skill a, they start learning skill b,
- etc.

For example, in a geometry class, students need to learn how to solve several different types of problems. For that purpose:

- they first spend several class periods learning how to solve problems of type a,

- then they spend several class periods learning how to solve problems of type b,
- etc.

Interleaving: an alternative approach. An alternative approach is *interleaving*, when students learn several skills at the same time. For example, instead of first solving several problems of type a, then several problem of type b, etc., they solve a problem of type a, then a problem of type b, etc., then again a problem of type a, then again b, etc.

In other words, instead of a usual sequence of problem types

$$aa \ldots abb \ldots bcc \ldots c \ldots,$$

we use an interleaving sequence

$$abc \ldots abc \ldots abc \ldots$$

Interleaving enhances learning. Several studies show that interleaving enhances different types of learning, from learning to play basketball [2, 4] to learning art [3] to learning mathematics [5, 6, 7]; see also [1].

Quantitative fact. In particular, in [7], it is shown that interleaving of four different types of geometric problems increases the average number of correct answers on the test twice, from 38% to 77%.

In other words, interleaving of four different types of problems doubles the learning speed.

What we do in this paper. In this paper, we provide a possible geometric explanation to the above enhancement.

2 A Possible Geometric Explanation

A simple geometric model. Let us describe traditional and interleaved approaches in geometric terms. We want students to learn to solve four different types of problems. In the beginning, the students do not know how to solve any of these problems. The objective is for them to be able to solve all four types of problems.

We can represent the state of the students at each moment of time by the percentage (x_1, x_2, x_3, x_4) of problems of each type that a student can solve.

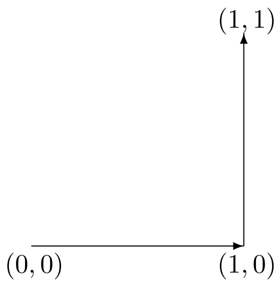
- In the beginning, the students are in the state $(0, 0, 0, 0)$.
- Our objective is to reach the state $(1, 1, 1, 1)$.

How traditional approach is represented in this geometric model. In the traditional approach, the students first learn to solve problems of the first type, then they learn how to solve problems of the second type, etc. In other words:

- the students first move from the state $(0, 0, 0, 0)$ to the state $(1, 0, 0, 0)$,
- then they move to the state $(1, 1, 0, 0)$,
- after that, they move to the state $(1, 1, 1, 0)$,
- and, finally, they move to the desired state $(1, 1, 1, 1)$.

At each stage of this process, we can assume that the students follow the shortest path – a straight line – to get to the corresponding state.

Each stage has length 1, so the total length of all four stages is equal to 4.



How the interleaved approach is represented in this geometric model. In the interleaved approach, at each moment of time, the students have spent equal time on problems of all fours types and thus, their skills in solving problems of all four types are equal.

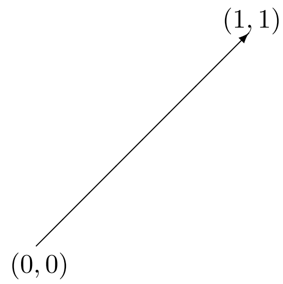
In geometric terms, this means that their state is described by a tuple (x, x, x, x). Thus, for this approach, learning follows the diagonal path

$$\{(x, x, x, x) : x \in [0, 1]\}.$$

This diagonal is the straight line segment connecting the original state $(0, 0, 0, 0)$ with the desired state $(1, 1, 1, 1)$.

The length of this path is equal to the distance between these two states $(0, 0, 0, 0)$ and $(1, 1, 1, 1)$, i.e., to the value

$$\sqrt{(1-0)^2 + (1-0)^2 + (1-0)^2 + (1-0)^2} = \sqrt{4} = 2.$$



Resulting explanation of the empirical fact. We see that in the interleaved approach, the path to the desired state is *twice shorter* than in the traditional approach. This may explain why, when we interleave four different types of problems, learning becomes *twice faster*.

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