Increased Climate Variability Is More Visible Than Global Warming: A General System-Theory Explanation

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Abstract

While global warming is a statistically confirmed long-term phenomenon, its most visible consequence is not the warming itself but, somewhat surprisingly, the increased climate variability. In this paper, we use general system theory ideas to explain why increased climate variability is more visible than the global warming itself.

1 Formulation of the Problem

What is global warming. The term "global warming" usually refers to the fact that there is a statistically significant long-term increase in the average temperature; see, e.g., [1, 2, 3, 4].

Somewhat surprisingly, what we mainly observe is not global warning itself, but rather related climate variability. Researchers have analyzed the expected future consequences of global warming: increase in temperature, melting of glaciers, raising sea level, etc. A natural hypothesis was that at present, when the effects of global warming are just starting, we would see the same effects, but at a smaller magnitude. This turned out not to be the case.

Some places do have the warmest summers and the warmest winters in record. However, other places have the coldest summers and the coldest winters on record.

What we do observe in all these cases is not so much the direct effects of the global warming itself, but rather an increased climate variability, an increase not so much in the *average* temperatures but rather in the *variance* of the temperature: in both unusually warm days and unusually cold days, what we observe is unusually high deviations from the average.

This phenomenon is sometimes called *climate change*, but a more proper description should be *increased climate variability* [1, 2, 3, 4].

Why is increased climate variability more visible than global warming? A natural question. A natural question is: why is increased climate variability more visible than global warming – which is supposedly the reason for this increased variability?

A usual answer to this question – and its limitations. A usual answer to the above question is that the increased climate variability is what computer models predict. However, the existing models of climate change are still very crude. Their quantitative predictions are usually very approximate and often unreliable, even on the qualitative scale: for example, none of these models explains the fact that the growth in the average temperature has drastically slowed down in the last two decades [1, 2, 3, 4].

It is therefore desirable to supplement the usual computer-model-based answer to the above question by more reliable explanations.

What we do in this paper. In this paper, we show that, on the qualitative level, the fact that the increase climate variability is more observable than the global warming can be explained in general system-theoretic terms.

2 Towards Formulation of the Problem in Precise Terms: A Simplified System-Theory Model

Towards a simplified model: first approximation. For simplicity, let us consider the simplest possible model, in which the state of the Earth is described by a single parameter x. In our case, x can be an average Earth temperature or the temperature at a certain location.

We want to describe how the value x of this parameter changes with time. In other words, we want to describe the derivative $\frac{dx}{dt}$.

There are external forces affecting the dynamics. So, in the first approximation, we can say that $\frac{dx}{dt} = u(t)$, where $u(t)$ describes these external forces.

We know that, on average, these forces lead to a global warming, i.e., to the increase in the value of the parameter $x(t)$. In terms of our equation, this means that the average value of $u(t)$ is positive. Let us denote this average value by u_0, and the random deviation from this average by $r(t) \stackrel{\text{def}}{=} u(t) - u_0$, then $u(t) = u_0 + r(t)$.

For simplicity, we will assume that the random values $r(t)$ corresponding to different moments t are independent and identically distributed, with some standard deviation σ_0.

Towards the second approximation. The above simplified equation does not take into account the fact that most natural systems – including the system corresponding to climate – are somewhat resistant to change: if a system is not resistant to change, it would not have persisted in the presence of numerous external forces.

Resistance to change means that when a system deviates from its stable value x_0, forces appear that try to bring this system back to this stable value. From the mathematical viewpoint, this phenomenon is easier to describe if instead of the original variable x, we consider the difference $y \stackrel{\text{def}}{=} x - x_0$. In terms of this difference, when $y > 0$, we have a force that decreases y, and when $y < 0$, we have a force that increases y. When $y = 0$, i.e., when $x = x_0$, the system remains in the stable state, so there are no forces.

In precise terms, in the absence of external forces, the system's dynamics is described by an equation $\frac{dy}{dt} = f(y)$, where $f(0) = 0$, $f(y) < 0$ for $y > 0$, and $f(y) > 0$ for $y < 0$. Since the system is stable, deviations y from the stable state are relatively small, so we can expand $f(y)$ in Taylor series in y and retain only the first few terms in this expansion. In general, we have $f(y) = f_0 + f_1 \cdot y + \ldots$ The condition $f(0) = 0$ implies that $f_0 = 0$, so $f(y) = f_1 \cdot y + \ldots$ The condition that $f(y) < 0$ for $y > 0$ implies that $f_1 < 0$, i.e., that $f_1 = -k$ for some $k > 0$. Thus, by keeping only the leading term in the Taylor expansion, we get $f(y) = -k \cdot y$.

Thus, we arrive at the following equation.

Resulting equation.

$$\frac{dy}{dt} = -k \cdot y + u_0 + r(t). \tag{1}$$

Discussion. Due to the linear character of the equation (1), each solution of this equation can be represented as a sum $y(t) = y_s(t) + y_r(t)$ of the solutions $y_s(t)$ and $y_r(t)$ corresponding to the systematic (average) part u_0 of the outside force and and to the random part $r(t)$:

$$\frac{dy_s}{dt} = -k \cdot y_s + u_0; \tag{2}$$

$$\frac{dy_r}{dt} = -k \cdot y_r + r(t). \tag{3}$$

Here, the systematic component $y_s(t)$ describes the systematic change in temperature (global warming), while the random component $y_r(t)$ describes the random change in temperature, i.e., the increased climate variability.

An empirical fact that needs to be explained. We need to explain that, in spite of the fact that eventually, we will see the effects of the global warming itself, at present, the climate variability becomes more visible than the global warming itself. In other words, at present, the relative role $y_r(t)/y_s(t)$ of climate variability is much higher than it will be in the future, when the global warming may become significant.

How to describe this empirical fact in precise terms? The change in y is determined by two factors: the external force $u(t)$ and the parameter k

that describes how resistant is our system to this force (the larger k, the large resistance to the change).

While some part of global warming may be caused by the variations in Solar radiation, most climatologists agree that the prevailing part of the long-term global warming is caused by local processes – such as the greenhouse effect – that lower the system's natural resistance to changes. (What causes numerous debates is which proportion of the global warming is caused by human activities.)

Since the decrease in resistance is the major contribution to the observed phenomena, in the first approximation, we will consider only this decrease. In other words, we will assume that the forces remain the same, but the parameter k decreases with time.

In these terms, the observed phenomenon is that at present, when the resistance value k is still reasonably high, the ratio $y_r(t)/y_s(t)$ is much larger than it will be in the future, when the resistance k will decrease. In other words, what we need to explain is that this ratio decreases when the value k decreases.

When computing this ratio, we need to take into account that while the systematic component $y_s(t)$ is deterministic, the random component $y_r(t)$ is a random process, its values change wildly. To gauge the size of this random component, i.e., to gauge how far the random variable $y_r(t)$ deviates from 0, it is reasonable to use standard deviation $\sigma(t)$ of this random variable.

Thus, we arrive at the following formulation.

Resulting formulation of the problem. We fix values u_0 and σ_0. Then, for each k, we can form the solutions $y_s(t)$ and $y_r(t)$ of the differential equations (2) and (3) corresponding to $y_s(0) = 0$ and $y_r(0) = 0$, where $r(t)$ is a family of independent identically distributed random variables with 0 mean and standard deviation σ_0. Since $r(t)$ is random, the solution $y_r(t)$ is also random, so for each moment t, we can define the standard deviation $\sigma(t)$ of this solution.

We want to prove that for every moment t, for sufficiently large $k > 0$, when k decreases, then the ratio $\sigma(t)/y_s(t)$ also decreases.

3 Analysis of the Problem and the Main Result

Solving the equation for the systematic deviation $y_s(t)$. If we move all the terms containing the unknown $y_s(t)$ to the left-hand side, we get

$$\frac{dy_s(t)}{dt} + k \cdot y_s(t) = u_0. \tag{4}$$

For an auxiliary variable $z(t) \stackrel{\text{def}}{=} y_s(t) \cdot \exp(k \cdot t)$, we get

$$\frac{dz(t)}{dt} = \frac{dy_s(t)}{dt} \cdot \exp(k \cdot t) + y_s(t) \cdot \exp(k \cdot t) \cdot k = \exp(k \cdot t) \cdot \left(\frac{dy_s(t)}{dt} + k \cdot y_s(t) \right). \tag{5}$$

Thus, due to (4), we have

$$\frac{dz(t)}{dt} = u_0 \cdot \exp(k \cdot t). \tag{6}$$

We know that for $t = 0$, we have $y_s(0) = 0$ and thus, $z(0) = 0$. Thus, the value $z(t)$ can be obtained by integration:

$$z(t) = z(0) + \int_0^t u_0 \cdot \exp(k \cdot s)\, ds = u_0 \cdot \frac{\exp(k \cdot t) - 1}{k}. \tag{7}$$

Hence, for $y_s(t) = \exp(-k \cdot t) \cdot z(t)$, we get

$$y_s(t) = u_0 \cdot \frac{1 - \exp(-k \cdot t)}{k}. \tag{8}$$

Solving the equation for the random component $y_r(t)$. For the random component, we similarly get

$$y_r(t) = \exp(-k \cdot t) \cdot \int_0^t r(s) \cdot \exp(k \cdot s)\, ds. \tag{9}$$

The mean value of each variable $r(s)$ is 0, thus, the mean value $E[y_r(t)]$ of their linear combination $y_r(t)$ is also 0. Hence, the variance

$$\sigma^2(t) = E[(y_r(t) - E[y_r(t)])^2]$$

of the random component $y_r(t)$ is simply equal to the expected value $E[y_r^2(t)]$ of its square.

Due to the formula (9), we have

$$y_r(t)^2 = \exp(-2k \cdot t) \cdot \left(\int_0^t r(s) \cdot \exp(k \cdot s)\, ds \right) \cdot \left(\int_0^t r(v) \cdot \exp(k \cdot v)\, dv \right) =$$

$$\exp(-2k \cdot t) \cdot \int_0^t ds \int_0^t dv\, r(s) \cdot r(v) \cdot \exp(k \cdot s) \cdot \exp(k \cdot v). \tag{10}$$

Since the expected value of a linear combination is equal to the linear combination of expected values, we get

$$\sigma^2(t) = E[y_r(t)^2] =$$

$$\exp(-2k \cdot t) \cdot \int_0^t ds \int_0^t dv\, E[r(s) \cdot r(v)] \cdot \exp(k \cdot s) \cdot \exp(k \cdot v). \tag{11}$$

We assumed that the values $r(s)$ corresponding to different moments of time s are independent and identically distributed, with standard deviation σ_0. Thus, for $s \neq v$, we get $E[r(s) \cdot r(v)] = E[r(s)] \cdot E[r(v)] = 0$ and $E[r^2(s)] = \sigma_0^2$. Substituting these expressions into the formula (11), we conclude that

$$\sigma^2(t) = E[y_r(t)^2] = \exp(-2k \cdot t) \cdot \int_0^t ds\, \sigma_0^2 \cdot \exp(k \cdot s) \cdot \exp(k \cdot s) =$$

$$\exp(-2k \cdot t) \cdot \int_0^t \sigma_0^2 \cdot \exp(2k \cdot s)\, ds. \qquad (12)$$

This integral can be explicitly integrated, so we get

$$\sigma^2(t) = \sigma_0^2 \exp(-2k \cdot t) \cdot \frac{\exp(2k \cdot t) - 1}{2k} = \sigma_0^2 \cdot \frac{1 - \exp(-2k \cdot t)}{2k}. \qquad (13)$$

Analyzing the ratio. We are interested in the ratio $\sigma(t)/y_s(t)$ of two positive numbers. The value $\sigma(t)$ is the square root of the expression (13). To avoid the need to take square roots, we can take into account the fact that for positive numbers, the square function is increasing; thus, the desired ratio increases with the decrease in k if and only if its square

$$S(t) \stackrel{\text{def}}{=} \frac{\sigma^2(t)}{y_s^2(t)} \qquad (14)$$

increases. Let us thus analyze this new ratio $S(t)$.

Due to the formulas (8) and (13), we get

$$S(t) = \frac{\sigma_0^2}{u_0^2} \cdot \frac{(1 - \exp(-2k \cdot t)) \cdot k^2}{2k \cdot (1 - \exp(-k \cdot t))^2}. \qquad (15)$$

By using a known formula $a^2 - b^2 = (a - b) \cdot (a + b)$, we conclude that

$$1 - \exp(-2k \cdot t) = (1 - \exp(-k \cdot t)) \cdot (1 + \exp(-k \cdot t)). \qquad (16)$$

Substituting the expression (16) into the formula (15) and cancelling the terms k and $1 - \exp(-k \cdot t)$ in the numerator and in the denominator, we conclude that

$$S(t) = \frac{\sigma_0^2}{u_0^2} \cdot \frac{(1 + \exp(-k \cdot t)) \cdot k}{2 \cdot (1 - \exp(-k \cdot t))}. \qquad (17)$$

Conclusion. When the value k is reasonably large, we have $\exp(-k \cdot t) \approx 0$, thus,

$$S(t) \approx \frac{\sigma_0^2}{u_0^2} \cdot \frac{k}{2}. \qquad (18)$$

This ratio clearly decreases when k decreases. Thus, eventually, when the Earth's resistance k will decrease, this ratio will also decrease and so, we will start observing mainly the direct effects of global warming (as researchers originally conjectured) – unless, of course, we do something to prevent the negative effects of global warming.

Comment. In our analysis, we made a simplifying assumption that the climate system is determined by a single parameter x (or y). The conclusion, however, remains the same if we consider a more realistic model, in which the climate system is determined by several parameters $y_1, \ldots, y_n$.

Indeed, in this case, in our linear approximation, the dynamics is described by a system of linear differential equations

$$\frac{dy_i}{dt} = -\sum_{j=1}^{n} a_{ij} \cdot y_j(t) + u_i(t). \tag{19}$$

In the generic case, all eigenvalues λ_k of the matrix a_{ij} are different; in this case, the matrix can be diagonalized: by considering the linear combinations $z_k(t)$ corresponding to the eigenvectors, we get a system with the diagonal matrix a_{ij}, i.e., a system of the type

$$\frac{dz_k}{dt} = -\lambda_k \cdot z_k(t) + u_k(t). \tag{20}$$

For each of these equations, similar analysis enables us to reach the same conclusion – that the current ratio of the random to systematic effects is much higher than it will be in the future, when the effects of global warming will be larger.

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