Once We Know that a Polynomial Mapping Is Rectifiable, We Can Algorithmically Find a Rectification

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Problem. It is known that some polynomial mappings $\varphi : \mathbb{C}^k \to \mathbb{C}^n$ are *rectifiable* in the sense that there exists a polynomial mapping $\alpha : \mathbb{C}^n \to \mathbb{C}^n$ whose inverse is also polynomial and for which $\alpha(\varphi(z_1, \ldots, z_k)) = (z_1, \ldots, z_k, 0, \ldots, 0)$ for all $z_1, \ldots, z_k$. In many cases, the existence of such a rectification is proven indirectly, without an explicit construction of the mapping α.

Our first result. In this talk, we use Tarski-Seidenberg algorithm (for deciding the first order theory of real numbers) to design an algorithm that:

- given a polynomial mapping $\varphi : \mathbb{C}^k \to \mathbb{C}^n$ which is known to be rectifiable,
- returns a polynomial mapping $\alpha : \mathbb{C}^n \to \mathbb{C}^n$ that rectifies φ.

Our second result. The above general algorithm is not practical for large n, since its computation time grows faster than 2^{2^n}. To make computations more practically useful, for several important case, we have also designed a much faster alternative algorithm.