50 Years of Fuzzy Sets: Contributions to Fuzzy Theory (Preface to the special issue)

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Applications-motivated fuzzy theory is important. Exactly 50 years ago, in 1965, Lotfi A. Zadeh published a seminal paper [7] that started the field of fuzzy sets and systems. Since then, there have been thousands of efficient practical applications of fuzzy techniques; see, e.g., [4, 6] and references therein.

Now matter how great an idea, its practical applications are rarely straightforward and easy: there are challenges to overcome, choices to make. For complex practical problems, there are usually so many possible choices that it is not possible to find the right choice by an empirical trial-and-error, we need a theoretical guidance.

From this viewpoint, fuzzy applications are no exception: fuzzy theory is indeed behind many current successful applications of fuzzy techniques. Practical needs regularly generate new theoretical challenges, inspire new results in fuzzy theory – results that lead to new successful applications.

Successes and challenges of fuzzy theory. The 50th anniversary of fuzzy ideas is an excellent opportunity to provide an overview of significant results that have been achieved and propose potential new areas of applications and developments of fuzzy techniques. Lotfi Zadeh himself wrote, on the BISC mailing list, that there is a need to list "significant contributions of fuzzy theory to real-world applications".

This was the main motivation behind this special issue. Of course, listing all achievements and challenges would require a series of books, not just one issue of a journal, so we asked the authors to concentrate on the major results and challenges.

Typical application of fuzzy techniques: reminder. To better understand which are the major theoretical challenges facing fuzzy techniques, let us briefly recall a typical application of fuzzy techniques, such an application to control – and control applications are still the major application area for fuzzy.

A typical application starts with the expert knowledge, knowledge which experts formulate by using imprecise ("fuzzy") words from natural language, like "small" or "fast". The first stage of fuzzy techniques is converting such knowledge into the computer-understandable numerical terms. This is done by assigning membership or possibility degrees to different possible values of the corresponding quantities. As a result of this stage, we get several number-valued (fuzzy) functions, or, more generally, number-valued (fuzzy) relations.

It is worth mentioning that while in the original fuzzy methodology, the membership degrees and possibility degrees take values from the interval $[0, 1]$, since then, it has been shown that often, more general degrees (e.g., degrees which are themselves fuzzy numbers) lead to a more adequate representation of expert knowledge.

Once the corresponding fuzzy functions and relations are formed, on the second stage, we combine and otherwise process these functions and relations in line with the expert's rules. For example, in fuzzy control, for each premise of the type "if x is small and v is medium", we combine the degree to which x is small and the degree to which v is medium into a degree to which the premise as a whole is satisfied.

As a result of the second stage, we get *fuzzy* recommendations: we get degrees to which different control values are reasonable, we get degrees to which a given object belongs to different classes, etc. In many practical situations, we need to make a *crisp* decision, i.e., select one of the possible values. In such situation, we need to apply the their stage of fuzzy techniques, *defuzzification*, a transformation from fuzzy to crisp.

Related theoretical challenges and how they are covered in this special issue. While many empirically successful procedures have been developed for all three stages of fuzzy techniques, from the theoretical viewpoint, on all three stages, we still face important challenges.

On the first stage, the main challenge is to provide theoretical classification analysis of numerous semi-heuristic techniques for transforming imprecise expert knowledge into numbers. This analysis should enable us to better understand why different techniques are successful, and help us select a techniques which is the most appropriate for a given situation. In this issue, such an analysis is performed in a survey paper by D. Dubois and H. Prade [1], the paper that starts this issue. Some aspects of this problem are also handled in [5].

In regards to the second stage, researchers have performed a thorough analysis of different ways to combine and process different relations. For example, there are known classifications of "and"-operations (t-norms) and "or"-operations (t-conorms), classifications of uninorms, etc. The main remaining theoretical challenges, in our opinion, are related not so much to the (well-studied) *mathematics* of these operations, but rather to their (not as well-studied) *meaning.* Specifically, in contrast to physical quantities that have exact values, expert's degrees are only approximate: while an expert can usually distinguish between degrees 0.6 and 0.8, no expert can meaningfully distinguish between degrees 0.6 and 0.61. It is therefore important to make sure that all the operations with fuzzy relations are consistent with this similarity, in the sense that if we start with slightly different numerical degrees reflecting the same experts' opinions, the degrees obtained as a result of fuzzy processing should also be sufficiently similar, so that their meaning for the users should be the same. This challenge is analyzed in the second paper of this issue, paper [3] by A. Kheniche, B. De Baets, and L. Zedama.

On the third defuzzification stage, the main theoretical challenge is to explain why some defuzzification proceedures are empirically successful; this challenge is partly covered in [5].

An additional challenge is related to the above-mentioned fact that, to represent experts' knowledge more adequately, we need to go beyond $[0,1]$-valued degrees, e.g., to fuzzy-valued (type-2) degrees. In contrast to a reasonably well-studied $[0,1]$-based fuzzy logic, type-2 techniques are much less studied. A survey of recent theoretical results related to type-2 fuzzy is provided in the third paper of this volume, the paper [2] by J. Harding, C. Walker, and E. Walker. Another generalization, to general lattices, is handled in [3].

The final challenge comes from the fact that, surprisingly, fuzzy techniques – techniques originally designed to process expert knowledge – turned out to be often successful in situations where there is no expert knowledge, where all we have is data. While, as fuzzy researchers and practitioners, we can proud of these successes, from the theoretical viewpoint, these successes remain somewhat a mystery. The final paper of this volume [5] attempts to understand this mysterious success by providing theoretical foundations for the use of fuzzy techniques in no-expert-knowledge data processing.

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We also want to thank the readers, because it is for you the readers that this issue has been produced. We hope that the papers forming this special issue will help the readers come up with new interesting and useful theoretical results.

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