In Engineering Classes, How to Assign Partial Credit: From Current Subjective Practice to Exact Formulas (Based on Computational Intelligence Ideas)

Joe Lorkowski and
Vladik Kreinovich
Department of Computer Science
University of Texas at El Paso
El Paso, Texas 79968, USA
Emails: lorkowski@computer.org
vladik@utep.edu

Olga Kosheleva
Department of Teacher Education
University of Texas at El Paso
El Paso, Texas 79968, USA
Email: olgak@utep.edu

***Abstract*—When a student performed only some of the steps needed to solve a problem, this student gets partial credit. This partial credit is usually proportional to the number of stages that the student performed. This may sound reasonable, but in engineering education, this leads to undesired consequences: for example, a student who did not solve any of the 10 problems on the test, but who successfully performed 9 out of 10 stages needed to solve each problem will still get the grade of A ("excellent"). This may be a good evaluation of the student's intellectual ability, but for a engineering company that hires this A-level student, this will be an unexpected disaster. In this paper, we analyze this problem from the viewpoint of potential loss to a company, and we show how to assign partial credit based on such loss estimates. Our conclusion is that this loss (and thus, the resulting grade) depend on the size of the engineering company. Thus, to better understand the student's strengths, it is desirable, instead of a single overall grade, to describe several grades corresponding to different company sizes.**

I. Assigning Partial Credit: Formulation of the Problem

Need to assign partial credit.

- If on a test, a problem is solved correctly, then the student gets full credit.
- If the student did not solve this problem at all, or the proposed solution is all wrong, the student gets no credit for this problem.

In many cases, the student correctly performed some steps that leads to the solution, but, due to missing steps, still did not get the solution. To distinguish these cases from the cases when the student did not perform any steps at all, the student is usually given partial credit for this problem.

The more steps the student performed, the more partial credit this student gets. From the pedagogical viewpoint, partial credit is very important: it enables the student who is learning – but who has not yet reached a perfect-knowledge stage – to see his or her progress; see, e.g., [1], [3].

How partial credit is usually assigned. Usually, partial credit is assigned in a very straightforward way:

- if the solution to a problem requires n steps,
- and $k < n$ of these steps have been correctly performed,
- then we assign the fraction $\dfrac{k}{n}$ of the full grade.

For example:

- if a 10-step problem is worth 100 points and
- a student performed 9 steps correctly,
- then this student gets 90 points for this problem.

For engineering education, this usual practice is sometimes a problem.

- If our objective is simply to check intellectual progress of a student, then the usual practice of assigning partial credit makes perfect sense,
- However, in engineering education, especially in the final classes of this education, the goal is to check how well a student is prepared to take on real engineering tasks.

Let us show that from this viewpoint, the usual practice is not always adequate.

Let us consider a realistic situation when, out of 10 problems on the test, none of these problems were solved by the student. However, if

- in each of these ten problems,
- the student performed 9 stages out of 10,
- this student will get 9 points of the ten for this problem.

Thus, the overall number of points for the test is $10 \cdot 9 = 90$, and the student gets the grade of A ("excellent") for this test.

None of the original ten problem are solved, but still the student got an A. This does not sound right, especially when we compare it with a different student, who:

- correctly solved 9 problems out of 10, but
- did not even start solving the tenth problem.

This second student will also get the same overall grade of 90 out of 100 for this test. However, if we assume that this test simulates the real engineering situation, the second student clearly performed much better.

This example shows that for engineering education, we need to come up with a different scheme of assigning partial credit.

What we do in this paper. In this paper, we analyze the problem, and come up with an appropriate scheme for assigning partial credit.

II. Assigning Partial Credit in Engineering Education: Analysis of the Problem and the Resulting Formulas

Description of the situation. To appropriately analyze the problem, let us imagine that this student:

- has graduated and
- is working for an engineering company.

In such a situation, a natural way to gauge the student's skill level is to estimate the overall benefit that he or she will bring to this company.

Let us start with considering a single problem whose solution consists of n stages. Let us assume that the newly hired student can correctly perform k out of these n stages. This is not a test, this is a real engineering problem, it needs to be solved, so someone else must help to solve the remaining $n-k$ stages.

Possible scenarios. From the viewpoint of benefit to a company, it is reasonable to distinguish between two possible situations:

- It is possible that in this company, there are other specialists who can help with performing the remaining $n-k$ stages. In this case, the internal cost of this additional (unplanned) help is proportional to the number of stages.
- It is also possible that no such specialists can be found within a company (this is quite probable if we have a small company). In this case, we need to hire outside help. In such a situation, the main part of the cost is usually the hiring itself: the consultant needs to be brought in, and
  - the cost of bringing in the consultant
  - is usually much higher than the specific cost of the consultant performing the corresponding tasks.

Estimating expected loss to a company: case of a single problem. We want to gauge the student's grade based on the financial implication of his or her imperfect knowledge on the company that hires this student.

We cannot exactly predict these implications, since we do not know for sure whether the company will have other specialists in-house who can fill in the stages that the newly hired student is unable to perform. Thus, we have a situation of uncertainty. According to decision theory (see, e.g., [2], [6], [8]), in such situations, a reasonable decision maker should select an alternative with the largest value of expected utility. Thus, a reasonable way to gauge the student's effect on the company is to estimate the expected loss caused by the student's inability to perform some tasks.

To estimate this expected loss, we need to estimate:

- the losses corresponding to both above scenarios, and
- the probability of each of these scenarios.

As we have mentioned, in the first scenario, when all the stages are performed in-house, the cost is proportional to the number of stages. So, if we denote the cost of performing one stage in-house by c_i (i for "in-house"), the resulting cost is equal to $c_i \cdot (n-k)$.

In the second scenario, when we have to seek a consultant's help, as we also mentioned, the cost practically does not depend on the number of stages. Let us denote this cost by c_h (h for "help").

To complete the estimate, we need to know the probabilities of the two scenarios. Let p denote the probability that inside the company, there is a specialist that can help with performing a given stage. It is reasonable to assume that different stages are independent from this viewpoint, so the overall probability that we can find inside help for all $n-k$ stages is equal to p^{n-k}. Thus:

- with probability p^{n-k}, we face the first (in-house) scenario, with the cost $c_i \cdot (n-k)$;
- with the remaining probability $1-p^{n-k}$, we face the second (outside help) scenario, with the cost c_h.

The resulting expected loss is thus equal to

$$c_i \cdot p^{n-k} \cdot (n-k) + c_h \cdot \left(1 - p^{n-k}\right). \qquad (1)$$

Estimating expected loss to a company: case of several problems. Several problems on a test usually simulate different engineering situations that may occur in real life. We can use the formula (1) to estimate the loss caused by each of the problems.

Namely, let n be the average number of stages of each problem. Then, if in the j-th problem, the student can successfully solve k_j out of n problems, then the expected loss is equal to:

$$c_i \cdot p^{n-k_j} \cdot (n-k_j) + c_h \cdot \left(1 - p^{n-k_j}\right). \qquad (2)$$

The overall expected loss L can be then computed as the sum of the costs corresponding to all J problems, i.e., as the sum

$$L = \sum_{j=1}^{J} \left(c_i \cdot p^{n-k_j} \cdot (n - k_j) + c_h \cdot \left(1 - p^{n-k_j}\right) \right). \quad (3)$$

So how should we assign partial credit. Usually, the credit is counted in such as way that complete knowledge corresponds to 100 points, and a complete lack of knowledge corresponds to 0 points. In this case, to assign partial credit, we should subtract, from the ideal case of 100 point, an amount proportional to the expected loss caused by the student's lack of skills.

In other words, the grade g assigned to the student should be equal to

$$g = 100 - c \cdot L, \quad (4)$$

for an appropriate coefficient of proportionality c. The corresponding c should be selected in such a way that for a complete absence of knowledge, we should subtract exactly 100 points.

The complete lack of knowledge corresponds to the case when for each problem j, the student is not able to solve any stage, i.e., when $k_j = 0$ for all j. In this case, the formula (3) takes the form

$$\overline{L} = J \cdot (c_i \cdot p^n \cdot n + c_h \cdot (1 - p^n)). \quad (5)$$

We want to select the coefficient of proportionality c in such a way that this worse-case will be equal to 100: $c \cdot \overline{L} = 100$. From this equality, we conclude that

$$c = \frac{100}{\overline{L}} = \frac{100}{J \cdot (c_i \cdot p^n \cdot n + c_h \cdot (1 - p^n))}. \quad (6)$$

Substituting this expression and the expression (3) for L into the formula (4), we conclude that

$$g = 100 - 100 \cdot \frac{\sum_{j=1}^{J} \left(c_i \cdot p^{n-k_j} \cdot (n - k_j) + c_h \cdot \left(1 - p^{n-k_j}\right) \right)}{J \cdot (c_i \cdot p^n \cdot n + c_h \cdot (1 - p^n))}. \quad (7)$$

We can simplify this expression if we divide both numerator and denominator of this fraction by the factor c_h. In this case, this factor c_h disappears in terms proportional to c_h, and terms proportional to c_i become now proportional to the ratio

$$c'_i \stackrel{\text{def}}{=} \frac{c_i}{c_h} \ll 1.$$

As a result, we arrive at the following formula.

How to assign partial credit: the resulting formula. Our analysis shows that for a student who, for each j-th problems out of J, performed k_j out of n stages, should be given the following grade:

$$g = 100 - 100 \cdot \frac{\sum_{j=1}^{J} \left(c'_i \cdot p^{n-k_j} \cdot (n - k_j) + \left(1 - p^{n-k_j}\right) \right)}{J \cdot (c'_i \cdot p^n \cdot n + (1 - p^n))}. \quad (8)$$

Here c'_i is the ratio of an in-house cost of performing a stage to the cost of hiring an outside consultant.

III. Let Us Analyze the Resulting Formula for Assigning Partial Credit

Different types of companies. The above formula use a parameter: namely, the probability p that it is possible to perform a stage in-house, without hiring outside help. We have already mentioned that the value of this parameter depends on the company size:

- In a very big company, with many engineers of different type, this probability is close to 1.
- On the other hand, in a small company this probability is very small.

On these two extreme cases, let us illustrate the use our formula (8).

First extreme case: a very big company. In this case, when $p = 1$, we have $1 - p^{n-k_j} = 1 - p^n = 0$, so the formula (8) takes the simplified form

$$g = 100 - 100 \cdot \frac{\sum_{j=1}^{J} c'_i \cdot (n - k_j)}{J \cdot c'_i \cdot n}. \quad (9)$$

Dividing both numerator and denominator by $c'_i \cdot n$, we get

$$g = 100 - 100 \cdot \frac{\sum_{j=1}^{J} \left(1 - \frac{k_j}{n}\right)}{J}, \quad (10)$$

i.e., the formula

$$g = 100 \cdot \frac{1}{J} \cdot \sum_{j=1}^{J} \frac{k_j}{n}. \quad (11)$$

This is the usual formula for assigning partial credit – thus, this formula corresponds to the case when a student is hired by a very big company.

Second extreme case: a very small company. In this case, $p = 0$, i.e., every time a student is unable to solve the problem, the company has to hire an outside help. The cost of outside help does not depend on how many stages the student succeeded in solving: whether a student performed all but one stages or none, the loss is the same.

In this case, the student gets no partial credit at all – if the answer is not correct, there are 0 points assigned to this student.

Comment. This situation is similar to how grades are estimated on the final exams in medicine: there, an "almost correct" (but wrong) answer can kill the patient, so such "almost correct" answers are not valued at all.

Intermediate case: general description. In the intermediate cases, we do assign some partial credit, but this credit is much smaller than in the traditional assignment (which, as we have shown, corresponds to the large-company case).

Intermediate case: towards an approximate formula. We have already mentioned that $c_i \ll c_h$ and thus, $c'_i \ll 1$. Thus, we can safely ignore the terms proportional to c_i' in our formula (8). As a result, we get the following approximate formula:

$$g = 100 \cdot \left(1 - \frac{\sum\limits_{j=1}^{J} \left(1 - p^{n-k_j}\right)}{J \cdot (1 - p^n)}\right).$$

If we subtract the fraction from 1, we get

$$g = 100 \cdot \frac{J \cdot (1 - p^n) - \sum\limits_{j=1}^{J} \left(1 - p^{n-k_j}\right)}{J \cdot (1 - p^n)},$$

i.e.,

$$g = 100 \cdot \frac{1}{J} \cdot \sum_{j=1}^{J} \frac{p^{n-k_j} - p_n}{1 - p^n}. \quad (12)$$

In other words, the overall grade for the test is equal to the average of the grades g_j for all the problems:

$$g = 100 \cdot \frac{1}{J} \cdot \sum_{j=1}^{J} g_j, \quad (13)$$

where

$$g_j \stackrel{\text{def}}{=} \frac{p^{n-k_j} - p_n}{1 - p^n}. \quad (14)$$

Usually, the number of stages p is reasonably large, so $p^n \approx 0$, and we arrive at the following formulas.

Intermediate case: resulting approximate formulas. The grade for the text can be described as an average of the grades g_j for individual problems, where for each problem, if a student performed k_j steps out of n, the grade is:

$$g_j \approx p^{n-k_j}. \quad (15)$$

Comment. A similar formula can be obtained if we consider a more general case, when different problems j may have different stages n_j. In this case, the general formula has the form

$$g = 100 -$$

$$100 \cdot \frac{\sum\limits_{j=1}^{J} \left(c_i \cdot p^{n_j-k_j} \cdot (n_j - k_j) + c_h \cdot \left(1 - p^{n_j-k_j}\right)\right)}{\sum\limits_{j=1}^{J} \left(c_i \cdot p^{n_j} \cdot n_j + c_h \cdot (1 - p^{n_j})\right)}, \quad (16)$$

and the corresponding approximate formula reduced to the average of the grades

$$g_j \approx p^{n_j-k_j}. \quad (17)$$

IV. So What Do We Propose

Our analysis shows that, depending on the size of company, we should assign partial credit differently. So maybe this is a way to go:

- instead of trying to describe the student's knowledge by a single number,
- we use different numbers corresponding to several different values of the parameter p (i.e., in effect, depending on the size of the company).

For example, we can select values $p = 0, p = 0.1, \ldots, p = 0.9$, and $p = 1$. Or, as a start, we can select just values $p = 0$ and $p = 1$ (maybe supplemented with $p = 0.5$).

We recommend that all these grades should be listed in the transcript, and each company should look into the grade that is the best fit for their perceived value p.

Comment. In this paper, we assumed that the student is absolutely confident about his or her answers. In real life, the student is often not fully confident. The corresponding degree of confidence should also be taken into account when gauging the student's knowledge – i.e., when assigning partial credit; possible ways to take this uncertainty into account are described, e.g., in [5].

V. The Resulting Formulas for Partial Credit Are in Good Accordance with Computational Intelligence

Need for an intuitive interpretation of the above formula. Our formula for partial credit comes from a simplified – but still rather complicated – mathematical model. Since the model is simplified, we cannot be 100% sure that a more complex model would not lead to a different formula. To make this formula more acceptable, we need to supplement the mathematical derivation with a more intuitive explanation. Such an explanation will be provided in this section and in the following one; specifically:

- in this section, we will show that the above formula is in good accordance with the general well-accepted ideas of computational intelligence;
- in the following section, we show that this formula is also in good accordance with a common sense analysis of the situation.

Let us analyze the problem of partial credit from the viewpoint of computational intelligence. The grade for a problem can be viewed as the instructor's degree of confidence that the student knows the material. The only information that we can use to compute this degree is that:

- the student successfully performed k_j stages, and
- the student did not perform the remaining $n - k_j$ stages.

Let us start our analysis with the simplest case, when the problem has only one stage. In this case:

- if the student successfully performed this stage, then our degree of confidence in this student's knowledge is higher, while
- if the student did not perform this stage, our degree of confidence in this student's knowledge is lower.

Let D denote our degree of confidence in the student's knowledge when the student successfully performed the stage, and let d ($d < D$) be the degree of confidence corresponding to the case when the student did not success in performing this stage.

In general, each of the k_j successful stages add a confidence D, *and* each of $n - k_j$ unsuccessful stages add a confidence degree d. The need for combining different degrees of confidence is well-studied in fuzzy logic (see, e.g., [4], [7], [9]), where the degree of confidence in an "and"-statement $A \,\&\, B$ is estimated by applying an appropriate "and"-operation (a.k.a. t-norm) to the degrees of confidence in individual statements.

The two simplest (and most widely used) "and"-operations are $\min$ and product. Let us consider what will happen if we use each of these operations.

If we use the minimum "and"-operation, there will be no partial credit. Let us first analyze what will happen if we use $\min$ as the "and"-operation. In this case, we have only two possible results of applying this "and"-operation:

- if the student successfully performed all the stages of solving a problem, then the resulting degree of confidence is equal to
$$\max(D, \ldots, D) = D;$$
- on the other hand, if the student did not succeed in at least one stage, then, no matter how many stages were missed, we get
$$\min(D, \ldots, D, d, \ldots, d) = d.$$

So, in this case, we:

- either give the student the full credit – if the student has successfully performed all the stages,
- or, if the student failed in at least one stage of the problem, we give the student the exact same grade as if we failed in all the stages.

Thus, if we use the minimum "and"-operation, we do not assign any partial credits – and, as we have mentioned earlier, partial credit is pedagogically important.

What if we use the product "and"-operation. If we use the product "and"-operation, then the resulting degree of confidence is equal to

$$\mu = D \cdot \ldots \cdot D \ (k_j \text{ times}) \cdot d \cdot \ldots \cdot d \ (n - k_j \text{ times}) =$$
$$D_j^k \cdot d^{n-k_j}. \qquad (18)$$

As usual in fuzzy techniques, it is convenient to *normalize* these value, i.e., to divide all these degrees μ by the largest possible degree $\overline{\mu}$ – so that after the division, the largest of the new degrees $\mu' = \dfrac{\mu}{\overline{\mu}}$ will be equal to 1.

The largest degree is clearly attained when the student has successfully performed all n stages of solving the problem, i.e., when $k_j = n$ and, correspondingly, $n - k_j = 0$. In this case, formula (18) leads to $\overline{\mu} = D^n$. Dividing μ by $\overline{\mu}$, we conclude that

$$\mu' = \frac{\mu}{\overline{\mu}} = \frac{D_j^k \cdot d^{n-k_j}}{D^n} = \frac{d^{n-k_j}}{D^{n-k_j}} = p^{n-k_j}, \qquad (19)$$

where we have denoted $p \stackrel{\text{def}}{=} \dfrac{d}{D}$.

Conclusion: we get the exact same formula for partial credit. We can see that:

- the formula (19) that we obtained based on the ideas from computational intelligence, and
- the formula (17) that is obtained based on our mathematical model

are identical. Thus, our formula (17) is indeed in good accordance with the computational intelligence ideas.

VI. Commonsense Interpretation of Our Partial Credit Formula

Why we need such an interpretation. The justification that we provided in the previous chapter is based on the general description of different "and"-operations. These general descriptions have nothing to do with the specific problem of assigning partial credit. So maybe for this specific problem we should have used a different "and"-operation, which may lead us to different formula?

To make our formula for partial credit move convincing, it is therefore desirable to supplement that general justification with a commonsense interpretation which would be directly related to the problem of assigning partial credit.

Main idea behind about commonsense justification. Knowledge about each subject can be viewed as an ever-growing tree:

- We start with some very basic facts. We can say that these facts form the basic (first) level of the knowledge tree.
- Then, based on these very basic facts, we develop some concepts of the second level. For example, in Introduction to Computer Science, once the students understood the main idea of an algorithm, they usually start describing different types of variables: integers, real numbers, characters, strings, etc.
- Based on the knowledge of the second level, we then further branch out into different concepts of the third level, etc.

In the first approximation, we can assume that the branching b is the same on each level, so that:

- we have one cluster on concepts on Level 1,

- we have b different concepts (or clusters of concepts) on Level 2,
- we have b^2 different concepts or clusters of concepts on Level 3,
- ...
- we have b^{k-1} different concepts or clusters of concepts on each Level k,
- ..., and
- we have b^{n-1} different concepts or clusters of concepts on the highest level n.

The overall number of concepts of all levels from 1 to n is equal to the sum

$$1 + b + \ldots + b^{n-1} = \frac{b^n - 1}{b - 1}.$$

For each problem, usually, each of n stages corresponds to the corresponding level:

- the first stage corresponds to Level 1,
- the second stage corresponds to Level 2, ...
- the n-th stage corresponds to Level n.

From this viewpoint, when a student was able to only successfully perform k stages, this is an indication that this student have mastered only the concepts of the first k levels.

With this limited knowledge, out of all possible $\frac{b^n - 1}{b - 1}$ concepts, the student has mastered only

$$1 + b + \ldots + b^{k-1} = \frac{b^k - 1}{b - 1}$$

of them.

A reasonable way to gauge the student's knowledge is by estimating what is the portion of concepts that the student learned, i.e., by computing the ratio

$$\frac{\frac{b^k - 1}{b - 1}}{\frac{b^n - 1}{b - 1}} = \frac{b^k - 1}{b^n - 1}. \tag{20}$$

Here, n is reasonably large and k is also large – if k is the small, this means that the student is failing this class anyway, so it does not matter how exactly we assign partial credit. In this case, $b^n \gg 1$ and $b^k \gg 1$. So, by ignoring 1s in comparison with b^n and b^k, we can come up with the following approximate formula

$$\frac{b^k}{b^n} = \frac{1}{b^{n-k}} = p^{n-k}, \tag{21}$$

where we have denoted $p \stackrel{\text{def}}{=} \frac{1}{b}$.

Conclusion: we get the exact same formula for partial credit. We can see that:

- the formula (21) that we obtained based on our commonsense analysis if the partial credit problem, and
- the formula (17) that is obtained based on our mathematical model

are identical. Thus, our formula (17) is indeed in good accordance with common sense.

ACKNOWLEDGMENT

This work was also supported in part by the National Science Foundation grants HRD-0734825 and HRD-1242122 (Cyber-ShARE Center of Excellence) and DUE-0926721.

The authors are thankful to Nigel Ward for valuable discussions.

REFERENCES

[1] C. E. Beevers, D. G. Wild, G. R. McGuire, D. J. Fiddes, M. A. Youngson, "Issues of partial credit in mathematical assessment by computer", *Research in Learning Technologies*, 1999, Vol. 7, No. 1, pp. 26–32.

[2] P. C. Fishburn, *Utility Theory for Decision Making*, John Wiley & Sons Inc., New York, 1969.

[3] L. E. Holmes and L. J. Smith, "Student evaluations of faculty grading methods", *Journal of Education for Business*, 2003, Vol. 78, No. 6, pp. 318–323.

[4] G. Klir and B. Yuan, "Fuzzy Sets and Fuzzy Logic", Prentice Hall, Upper Saddle River, New Jersey, 1995.

[5] J. Lorkowski, O. Kosheleva, and V. Kreinovich, *How to Take Into Account a Student's Degree of Certainty When Evaluating the Test Results*, University of Texas at El Paso, Department of Computer Science, Technical Report UTEP-CS-15-34, 2015.

[6] R. D. Luce and R. Raiffa, *Games and Decisions: Introduction and Critical Survey*, Dover, New York, 1989.

[7] H. T. Nguyen and E. A. Walker, *A First Course in Fuzzy Logic*, Chapman and Hall/CRC, Boca Raton, Florida, 2006.

[8] H. Raiffa, *Decision Analysis*, Addison-Wesley, Reading, Massachusetts, 1970.

[9] L. A. Zadeh, "Fuzzy sets", *Information and Control*, 1965, Vol. 8, pp. 338–353.