Why We Need Extra Physical Dimensions: A Simple Geometric Explanation

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Abstract

It is known that a consistent description of point-wise particles requires that we add extra physical dimensions to the usual four dimensions of space-time. The need for such dimensions is based on not-very-intuitive complex mathematics. It is therefore desirable to try to come up with a simpler geometric explanation for this phenomenon. In this paper, we provide a simple geometric explanation of why extra physical dimensions are needed.

1 Need for Extra Physical Dimensions: Reminder

Problems with the usual 4-dimensional space-time models. In relativistic physics, elementary particles are points in space; see, e.g., [3]. Point-wise character of elementary particles makes many physical quantities infinite. For example, the energy density $\rho(x)$ of the electric field $\vec{E}(x)$ is known to be proportional to $|\vec{E}(x)|^2$, and the electric field of a point-wise particle decreases with the distance r to the particle according to the Coulomb law $|\vec{E}(x)| \sim \dfrac{1}{r^2}$. Thus, the energy density $\rho(x)$ is proportional to $|\vec{E}(x)|^2 \sim \dfrac{1}{r^4}$.

The overall energy is equal to the integral $\int \rho(x)\,dx$ and is, thus, proportional to the integral $\int \dfrac{1}{r^4}\,dx$. In polar coordinates, after integrating over angular coordinates, we get

$$I = \int_0^\infty \frac{2\pi \cdot r^2}{r^4}\,dr = 2\pi \cdot \int_0^\infty \frac{1}{r^2}\,dr.$$

This integral is equal to

$$I = -\,2\pi \cdot \frac{1}{r}\bigg|_0^\infty = \infty.$$

Similar physically meaningless infinities appear when we compute other quantities related to a point particle [3].

Comment. The above computations use a non-quantum approximation, but similar infinities appear when we take into account quantum effects as well.

Current solution. It turns out that infinities can be avoided if we assume that the space-time has extra dimensions beyond the four usual ones. For example, string theory shows that we can get a consistent physical theory if we assume that the space-time is 10-dimensional; see, e.g., [4].

Remaining challenge. A problem with this solution is that it is heavily mathematical, there is no simple intuitive geometric explanation of why extra dimensions are needed.

Comment. It should be mentioned that:

- while there is no clear geometric explanation of *why extra dimensions are needed*,
- there are simple geometric explanations of *why namely 10* is a good dimension; see, e.g., [5].

What we do in this paper. In this paper, we provide a possible geometric explanation of why extra space-time dimensions are needed.

2 Analysis of the Problem and the Resulting Explanation of Extra Physical Dimension(s)

Natural idea: discrete space-time. The infinities are caused by integration to $r = 0$. Thus, one possible way to avoid infinities is to assume that spatial coordinates – and other quantities – are discrete. This idea is ubiquitous in physics [3]:

- an electric charge cannot take any possible value, it must be proportional to some constant;
- quantum physics started with Planck's hypothesis that energy of light of a given wavelength cannot take any possible value, it must be proportional to some constant (dependent on this frequency), etc.

Resulting description of space-time. Let us apply the discreteness idea to variables that describe space-time geometry, namely,

- to the space-time coordinates $x_1, \ldots, x_n$, and

- to the components g_{ij} of the metric tensor that describes the proper time $s(x, x')$ between two points $x = (x_1, \ldots, x_n)$ and $x' = (x'_1, \ldots, x'_n)$ as follows:

$$s^2(x, x') = \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} \cdot (x_i - x'_i) \cdot (x_j - x'_j). \quad (1)$$

For space-time coordinates, discreteness means that all the coordinates must be integer multiples of some fixed quantum q_x, i.e., that for every point x and for each coordinate i, we must have $x_i = X_i \cdot q_x$ for some integer X_i. Similarly, for the components of the metric tensor g_{ij}, discreteness means that there exists some fixed quantum q_g for which, for each component g_{ij}, we have $g_{ij} = G_{ij} \cdot q_g$ for some integer G_{ij}.

Under these two discreteness assumptions, the formula (1) that describes the square $s^2(x, x')$ of the proper time between the points $x = (X_1 \cdot q_x, \ldots, X_n \cdot q_x)$ and $x' = (X'_1 \cdot q_x, \ldots, X'_n \cdot q_x)$ takes the form

$$s^2(x, x') = S^2(X, X') \cdot q_x^2 \cdot q_g, \quad (2)$$

where we denoted

$$S^2(X, X') \stackrel{\text{def}}{=} \sum_{i=1}^{n} \sum_{j=1}^{n} G_{ij} \cdot (X_i - X'_i) \cdot (X_j - X'_j). \quad (3)$$

Empirical fact: there are light-like particles. It is a known physical fact that:

- in addition to usual particles like electrons and protons that travel with speeds smaller than the speed of light, and for which, therefore, $s^2(x, x') > 0$ for every two points $x \neq x'$ on the particle's trajectory,
- there also exist "light-like" particles like photons that always travel with the speed of light and for which $s^2(x, x') = 0$ for every two points $x \neq x'$ on the particle's trajectory.

In the continuous space-time, the possibility of light-like particles is mathematically trivial. In the continuous space-time, when each coordinate x_i can take any real value, it is always possible to find pairs of points $x \neq x'$ for which $s^2(x, x') = 0$ – provided, of course, that the matrix g_{ij} is not positive or negative definite, i.e., provided that:

- there exist pairs (x, x') with $s^2(x, x') > 0$, and
- there exist pairs (x, x') with $s^2(x, x') < 0$.

In discrete space-time, the existence of light-like particles is automatically guaranteed only if we have extra physical dimensions. In the discrete space-time model (2)-(3), however, it is not always true that if a quadratic form (3) with integer coefficients G_{ij} attains both positive and negative values, there exist integer values $X_i - X_i'$ for which this form is equal to 0.

Such a general statement is true if and only if we have at least five variables, i.e., if and only if $n \geq 5$. This result was proven by A. Meyer in 1884 [6] and is known as Meyer's Theorem; see, e.g., [1, 7, 8].

Resulting explanation. Thus, to make sure that a discrete space-time is always consistent with the existence of light-like particles, we must assume that the dimension of space-time is at least five.

This explain the need for at least one extra physical dimension – in addition to the usual four dimensions of space-time.

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