Explaining Boris Pasternak's Observation that Complex Ideas Are Sometimes Easier to Understand

Olga Kosheleva and Vladik Kreinovich
University of Texas at El Paso
500 W. University
El Paso, TX 79968, USA
olgak@utep.edu, vladik@utep.edu

Abstract

Probably the most cited lines from the poetry of the Nobel-prize winning Russian writer Boris Pasternak contain the observation that complex ideas are sometimes easier to understand than simpler ones. This is not just a paradoxical poetic statement: many teachers have observed the same seemingly counter-intuitive phenomenon. In this paper, we provide a possible explanation for this phenomenon, by showing that indeed, many easier-to-describe mathematical models lead to more-difficult-to-solve mathematical problems.

1   Formulation of the Problem

Complex ideas are easier to understand: a somewhat counter-intuitive observation. One of the most popular poems of the Nobel Prize winning writer Boris Pasternak, titled *The Waves*, has the lines that many Russian-speaking people know by heart: "Nel'zia ne vpast' k kontsu, kak v eres', v neslyhannuiu prostotu. No my poschazeny ne budem, Kogda ee ne utaim. One vsego nuzhnee liudiam, No slozhnoe poniatnei im." The meaning of these lines, written in 1931, is well preserved in the following English translation from [3]:

Assured of kinship with all things
And with the future closely knit
We can't but fall - a heresy! -
To unbelievable simplicity.

But to be spared we can't expect
If we do not conceal it closely.
Men need it more than anything,

But complex things are easier for them ...

This is not just an exotic poetic observation, this observation is in good accordance with the experience of many teachers: that often, students understand complex ideas much easier than simple ones.

Problem. Intuitively, one should expect exactly the opposite: that the simpler the idea, the easier it should be for the students to understand it. So why, contrary to this intuition, do we often observe an opposite phenomenon?

This is the question that we will try to answer in this paper.

2   Our Explanation

Main idea of our explanation. Our explanation for the above observation is that learning a concept means ability to deal with this concept, to solve the corresponding problems.

And, as we will show, among basic mathematical models, easier-to-formulate ideas are indeed more difficult to analyze – and lead to more difficult problems.

Basic mathematical models: reminder. Basic mathematical models are usually described in terms of real numbers.

Which basic models are simpler to describe. The simplest case is when these numbers are integers: for example, it is easier to deal with counting objects than with measuring them.

Which basic models are the easiest to solve: common sense intuition. In general, the more options we add, the more difficult it is to choose one of these options. In particular, when we go from integers to real numbers, then we add numerous new options and so, it seems like we should expect the corresponding problems to become more complex.

Somewhat surprisingly, the reality is different: problems related to real numbers are, in general, easier to solve than problems related to integers.

First example: general problems related to real numbers are algorithmically decidable, while general problems related to integers are not algorithmically decidable. Let us first consider the general class of all possible problems related to real numbers or integers.

In a general problem, we want to check whether a certain property is true, or find a value that satisfies a given property. Properties can be complex, they may involve quantifiers "for all" and "there exists". Such general properties are known as *properties from the first order logic*, or *first order properties*, for short. Let us give a formal definition:

- we start with *variables* $x_1, \ldots, x_n, \ldots$ that can take any values either from real numbers or from integers;

- by a *term*, we mean any string constructed from variables and integers by using four arithmetic operations;

- by an *elementary formula*, we mean an expression of the type $t_1 = t_2$, $t_1 > t_2$, $t_1 < t_2$, $t_1 \geq t_2$, $t_1 \leq t_2$, or $t_1 \neq t_2$, where t_i are terms;

- finally, a *formula* is any expression that can be obtained from elementary formulas by using propositional connectives & ("and"), $\vee$ ("or"), $\neg$ ("not"), $\rightarrow$ ("implies"), and quantifiers $\forall x_i$ and $\exists x_i$.

For example, the statement that any non-negative real number has a square root can be described by the following formula:

$$\forall x_1 \, (x_1 \geq 0 \rightarrow \exists x_2 \, (x_1 = x_2 \cdot x_2)).$$

It is interesting to mention that for real numbers, there is an algorithm that, given such a formula, tells whether this formula is true or not. This algorithm was discovered by Alfred Tarski in the late 1940s [1, 4]. On the other hand, for formulas involving integers, no such algorithm is possible – this is, in effect, the well-known Gödel's theorem proven in the early 1930s; see, e.g., [2].

Second example: if we limit ourselves to problems with finite choice, then real-valued problems are feasible while a similar integer-valued problem are, in general, NP-hard. What if we limit ourselves to situations when we have finitely many choices, so that all the problems become algorithmically decidable – e.g., by trying all possible choices.

For such problems, the question is how many computational steps we need to solve the corresponding problem. Here, also, there is a big difference between real-valued problems and similar integer-values problems.

For example, if we simply want to solve a system of linear equations under bounds on x_i, then, in case of real numbers, there are known feasible algorithms – e.g., Gauss elimination, while the corresponding integer-valued problem is known to be NP-hard [2]. Moreover, it is known that even finding a solution a bounded integer solution to a single linear equation is NP-hard: namely, if we are given natural numbers $s_1, \ldots, s_n, S$, then the problem of finding integers $x_i \in \{0, 1\}$ for which $\sum_{i=1}^{n} x_i \cdot s_i = S$ is NP-hard [2], i.e., much more complex than the corresponding real-valued problem.

Third example: going beyond real numbers. If we go beyond real numbers – e.g., towards complex numbers, then some problems become even easier. For example, for real-valued polynomials of one variable, we need a rather complex algorithm to decide whether this polynomial has a root, it is known that every non-constant complex-valued polynomial has a complex root – so this property is easier to detect for complex numbers.

Conclusion. These examples explain Boris Pasternak's observation that often, easier-to-describe (and seemingly either-to-solve) concepts leads to much-more-difficult-to-solve problems.

Acknowledgments

References

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