How to Make a Solution to a Territorial Dispute More Realistic: Taking into Account Uncertainty, Emotions, and Step-by-Step Approach

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Dividing a disputed territory: a real-life problem. In many real-life situations, from conflicts between neighboring farms to conflict between states, there is a dispute over a territory, as a result of which none of the sides can use this territory efficiently. In such situations, it is desirable to come up with a mutually beneficial agreement.

Current solution. The current solution is based on the work by the Nobelist J. Nash, who showed that under reasonable assumptions, the best mutually beneficial solution is the one that maximizes the product of utilities of all the sides [3]. Let $u_i(x)$ be the utility (per area) of the i-th participant at location x. Then, we should select a partition for which the product $\prod_{i=1}^{n} U_i$ is the largest, where $U_i \stackrel{\text{def}}{=} \int_{S_i} u_i(x)\,dx$ and S_i is the set allocated to the i-th participant. The solution to this optimization problem is, for some thresholds t_i, to assign each location x to a participant with the largest ratio $u_i(x)/t_i$; see, e.g., [4]. In particular, for two participants, $x \in S_1$ if $u_1(x)/u_2(x) \geq t \stackrel{\text{def}}{=} t_1/t_2$.

How to make this solution more realistic. (1) In reality, we know the values $u_i(x)$ only approximately; e.g., we only know the interval $[\underline{u}_i(x), \overline{u}_i(x)]$ containing $u_i(x)$. In this case, decision making theory recommends using Hurwicz's optimism-pessimism utilities $\alpha_i \cdot \overline{u}_i(x) + (1-\alpha_i) \cdot \underline{u}_i$, where $\alpha_i \in [0,1]$ describe the i-th participant's degree of optimism. (2) Because of emotional involvement, the actual utility U_i^{act} can also involve gains of others $U_i^{\text{act}} = U_i + \sum_j \alpha_{ij} \cdot U_j$. (3) Finally, while the above formula proposes an immediate solution, participants often follow step-by-step approach, where they first divide a small part, then another part, etc.

In this talk, we describe how to take all this into account; preliminary results appeared in [1, 2].

References

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