

# How to Estimate Resilient Modulus for Unbound Aggregate Materials: A Theoretical Explanation of an Empirical Formula

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**Formulation of the problem.** To ensure the quality of a road, it is important to make sure that all the pavement layers have reached a certain stiffness level. To characterize stiffness of unbound pavement materials, transportation engineers use *resilient modulus*  $M_r$ . A material's resilient modulus is actually an estimate of its modulus of elasticity  $E$ , i.e., of ratio of stress by strain; the difference from the usual modulus of elasticity is that the usual modulus corresponds to a *slowly* applied load, while the resilient characterizes the effect of *rapidly* applied loads – like those experienced by pavements. A precise definition of the resilient modulus is given, e.g., in [1].

In the usual (*linear*) elastic materials, the modulus does not depend on the stress value. In contrast, pavement materials are usually *non-linear*, in the sense that the resilient stress depends on the stress. Several empirical formulas have been proposed to describe this dependence. Experimental comparison [2] shows that the best description is provided by the formula (first proposed in [3])  $M_r = k_1' \cdot \left( \frac{\theta}{P_a} + 1 \right)^{k_2'} \cdot \left( \frac{\tau_{\text{oct}}}{P_a} + 1 \right)^{k_3'}$ , where  $P_a$  is atmospheric pressure,  $\theta$  is the *bulk stress*, i.e., the trace  $\theta = \sum_{i=1}^3 \sigma_{ii}$  of the stress tensor  $\sigma_{ij}$  (see, e.g., [4]), and  $\tau_{\text{oct}} \stackrel{\text{def}}{=} \sqrt{\frac{1}{3} \cdot \sum_{ij} \sigma_{ij}^2 - \frac{1}{3} \cdot \theta^2}$  is the *octahedral shear stress*.

**What we do in this talk.** In this talk, we provide a theoretical explanation for the above empirical formula. This explanation uses the general idea that the fundamental physical formulas should not change if we simply changing the measuring unit and/or the starting point for the measurement scale.

## References

- [1] American Association of State Highway and Transportation Officials (AASHTO), “Resilient Modulus of Subgrade Soils and Untreated Base/Subbase Materials”, Standard T 292-91.
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- [3] P. S. K. Ooi, A. R. Archilla, and K. G. Sandefur, “Resilient modulus models for comactive cohesive soils”, *Transportation Research Record*, 2006, No. 1874, pp. 115–124.
- [4] M. H. Sadd, *Elasticity: Theory, Applications, and Numerics*, Academic Press, Oxford, UK, and Waltham, Massachusetts, 2014.