Beyond Traditional Applications of Fuzzy Techniques: Main Idea and Case Studies

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Fuzzy logic techniques were originally designed to translate expert knowledge -- which is often formulated by using imprecise ("fuzzy") from natural language (like "small") -- into precise computer-understandable models and control strategies. Such a translation is still the main use of fuzzy techniques. For example, we want to control a complex plant for which no good control technique is known, but for which there are experts how can control this plant reasonably well. So, we elicit rules from the experts, and then we use fuzzy techniques to translate these rules into a control strategy.

Lately, it turned out that fuzzy techniques can help in another class of applied problems: namely, in situations when there are semi-heuristic techniques for solving the corresponding problems, i.e., techniques for which there is no convincing theoretical justification. Because of the lack of a theoretical justification, users are reluctant to use these techniques, since their previous empirical success does not guarantee that these techniques will work well on new problems.

Also, these techniques are usually not perfect, and without an underlying theory, it is not clear how to improve their performance. For example, linear models can be viewed as first approximation to Taylor series, so a natural next approximation is to use quadratic models. However, e.g., for $l^p$-models, when they do not work well, it is not immediately clear what is a reasonable next approximation.

In this talk, we show that in many such situations, the desired theoretical justification can be obtained if, in addition to known (crisp) requirements on the desired solution, we also take into account requirements formulated by experts in natural-language terms. Naturally, we use fuzzy techniques to translate these imprecise requirements into precise terms. To make the resulting justification convincing, we need to make sure that this justification works not only for one specific choice of fuzzy techniques (i.e., membership function, "and"- and "or"-operations, etc.), but for all combinations of such techniques which are consistent with the corresponding practical problem.

As examples, we provide a justification of:

* sparsity techniques in data and image processing -- a very successful hot-topic technique whose success is often largely a mystery;

* $l^p$-regularization techniques in solving inverse problems -- an empirically successful alternative to so-called Tikhonov regularization appropriate for situations when the desired signal or image is not smooth.