

# Why Compaction Meter Value (CMV) Is a Good Measure of Pavement Stiffness: Towards a Possible Theoretical Explanation

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## Abstract

To measure stiffness of the compacted pavement, practitioners use the Compaction Meter Value (CMV); a ratio between the amplitude for the first harmonic of the compactor's acceleration and the amplitude corresponding to the vibration frequency. Numerous experiments show that CMV is highly correlated with the pavement stiffness, but as of now, there is no convincing theoretical explanation for this correlation. In this paper, we provide a possible theoretical explanation for the empirical correlation. This explanation also explains why, the stiffer the material, the more higher-order harmonics we observe.

## 1 Compaction Meter Value (CMV) – An Empirical Measure of Pavement Stiffness

**Need to measure pavement stiffness.** Road pavement must be stiff: the pavement must remain largely unchanged when heavy vehicles pass over it.

To increase the pavement's stiffness, pavement layers are usually compacted by the rolling compactors. In the cities, only non-vibrating compactors are used, to avoid human discomfort caused by vibration. However, in roads outside the city limits, vibrating compactors are used, to make compaction more efficient. In this paper, we will denote the vibration frequency by  $f$ .

Compaction is applied both to the soil and to the stiffer additional pavement material that is usually placed on top of the original soil. To check whether we

need another round of compaction and/or another layer of additional material on top, we need to measure the current pavement stiffness.

**Ideally, we should measure stiffness as we compact.** In principle, we can measure stiffness *after* each compaction cycle, but it would be definitely more efficient to measure it *during* the compaction – this way we save time and we save additional efforts needed for post-compaction measurements.

What we *can* rather easily measure during compaction is acceleration; it is therefore desirable to estimate the pavement stiffness based on acceleration measurements.

**Compaction Meter Value (CMV).** It turns out that reasonably good estimates for stiffness can be obtained if we apply Fourier transform to the signal describing the dependence of acceleration on time, and then evaluate *Compaction Meter Value* (CMV), a ratio  $A_2/A_1$  between the amplitudes corresponding to the frequencies  $2f$  and  $f$ . This measure was first introduced in the late 1970s [3, 10, 11].

Numerous experiments have confirmed that CMV is highly correlated with more direct characteristics of stiffness such as different versions of elasticity modulus; see, e.g., [2, 6, 7, 12, 13].

CMV remains one of the main ways of estimating stiffness; see, e.g., [5].

**Can we use other Fourier components?** Since the use of the double-frequency component turned out to be so successful, a natural idea is to try to use other Fourier components.

It turns out that when the soil is soft (not yet stiff enough), then even the double-frequency Fourier component is not visible above noise. As the pavement becomes stiffer, we can clearly see first the first harmonic, then also higher harmonics, i.e., harmonics corresponding to  $3f$ ,  $4f$ , etc.

**Remaining problem.** While the relation between CMV and stiffness is an empirical fact, from the theoretical viewpoint it remains somewhat a mystery: to the best of our knowledge, there is no theoretical explanation for this empirical dependence.

In this paper, we attempt to provide such a theoretical explanation.

## 2 A Possible Theoretical Explanation of an Empirical Correlation Between CMV and Stiffness

**Analysis of the problem: towards the corresponding equations.** Let us start our analysis with the extreme situation when there is no stiffness at all. Crudely speaking, the complete absence of stiffness means that particles forming the soil are completely independent from each one other: we can move some of them without affecting others.

In this extreme case, the displacement  $x_i$  of each particle  $i$  is determined by the Newton's equations

$$\frac{d^2 x_i}{dt^2} = \frac{1}{m_i} \cdot F_i, \quad (1)$$

where  $m_i$  is the mass of the  $i$ -th particle and  $F_i$  is the force acting on this particle. For a vibrating compactor, the force  $F_i$  is sinusoidal with frequency  $f$ . Thus, the corresponding accelerations are also sinusoidal with this same frequency. In this extreme case, after the Fourier transform, we will get only one component – corresponding to the vibration frequency  $f$ .

Stiffness  $k$  means that, in addition to the external force  $F_i$ , the acceleration of each particle  $i$  is also influence by the locations of other particles  $x_j$ . For example, if we move one of the particles forming the soil, other particle move as well so that the distances between the particles remain largely the same. Thus, instead of the simple Newton's equations (1), we have more complicated equations

$$\frac{d^2 x_i}{dt^2} = \frac{1}{m_i} \cdot F_i + f_i(k, x_1, \dots, x_N), \quad (2)$$

for some expression  $f_i(k, x_1, \dots, x_N)$ .

Displacements are usually small. We consider the case when stiffness is also reasonably small. It is therefore reasonable to expand this expression in Taylor series and keep only the first few terms in this expansion.

With respect to  $k$ , in the first approximation, we just keep linear terms. With respect to  $x_j$ , it is known that the corresponding processes are observably non-linear (see, e.g., [1, 4, 9]) so we need to also take non-linear terms into account; the simplest non-linear terms are the quadratic ones, so we end up with the following approximate model:

$$\frac{d^2 x_i}{dt^2} = \frac{1}{m_i} \cdot F_i + k \cdot \sum_{j=1}^N a_{ij} \cdot x_j + k \cdot \sum_{j=1}^N \sum_{\ell=1}^N a_{ij\ell} \cdot x_j \cdot x_\ell. \quad (3)$$

**Solving the resulting equations.** In general, the solution to the equations (3) depends on the value  $k$ :  $x_i(t) = x_i(k, t)$ .

When deriving the equations (3), we ignored terms which are quadratic (or of higher order) in terms of  $k$ . It is therefore reasonable, when looking for solutions to this equation, to also ignore terms which are quadratic (or of higher order) in  $k$ , i.e., to take

$$x_i(k, t) = x_i^{(0)}(t) + k \cdot x_i^{(1)}(t). \quad (4)$$

If we plug in the formula (5) into the equation (3) and ignore terms which are quadratic in  $k$ , then we end up with the equation

$$\frac{d^2 x_i^{(0)}}{dt^2} + k \cdot \frac{d^2 x_i^{(1)}}{dt^2} = \frac{1}{m_i} \cdot F_i + k \cdot \sum_{j=1}^N a_{ij} \cdot x_j^{(0)} + k \cdot \sum_{j=1}^N \sum_{\ell=1}^N a_{ij\ell} \cdot x_j^{(0)} \cdot x_\ell^{(0)}. \quad (5)$$

This formula should hold for all  $k$ , so:

- terms independent on  $k$  should be equal on both sides, and
- terms linear in  $k$  should be equal on both sides.

By equating terms in (5) that do not depend on  $k$ , we get the linear equation

$$\frac{d^2 x_i^{(0)}}{dt^2} = \frac{1}{m_i} \cdot F_i, \quad (6)$$

which, for the sinusoidal force  $F_i(t) = A_i \cdot \cos(\omega \cdot t + \Phi_i)$ , has a similar sinusoidal form

$$x_i^{(0)}(t) = a_i \cdot \cos(\omega \cdot t + \varphi_i) \quad (7)$$

for appropriate values  $a_i$  and  $\varphi_i$ .

By equating terms linear in  $k$  on both sides of the equation (5), we conclude that

$$\frac{d^2 x_i^{(1)}}{dt^2} = \sum_{j=1}^N a_{ij} \cdot x_j^{(0)} + \sum_{j=1}^N \sum_{\ell=1}^N a_{ij\ell} \cdot x_j^{(0)} \cdot x_\ell^{(0)}. \quad (8)$$

For the sinusoidal expression (7) for  $x_i^{(0)}$ :

- linear terms  $\sum_{j=1}^N a_{ij} \cdot x_j^{(0)}$  in the right-hand side are sinusoidal with the same angular frequency  $\omega$  (i.e., with frequency  $f$ ), while
- quadratic terms  $\sum_{j=1}^N \sum_{\ell=1}^N a_{ij\ell} \cdot x_j^{(0)} \cdot x_\ell^{(0)}$  are sinusoids with the double angular frequency  $2\omega$  (i.e., with double frequency  $2f$ ).

Thus, the right-hand side of the equation (8) is the sum of two sinusoids corresponding to frequencies  $f$  and  $2f$ , and so,

$$\begin{aligned} \frac{d^2 x_i}{dt^2} &= \frac{d^2 x_i^{(0)}}{dt^2} + k \cdot \frac{d^2 x_i^{(1)}}{dt^2} = A_i \cdot \cos(\omega \cdot t + \Phi_i) + \\ &k \cdot \left( A_i^{(1)} \cdot \cos(\omega \cdot t + \Phi_i^{(1)}) + A_i^{(2)} \cdot \cos(2\omega \cdot t + \Phi_i^{(2)}) \right). \end{aligned} \quad (9)$$

The measured acceleration  $a(t)$  is the acceleration of one of the points  $a(t) = \frac{d^2 x_{i_0}(t)}{dt^2}$ , thus the measured acceleration has the form

$$\begin{aligned} a(t) &= A_{i_0}^{(0)} \cdot \cos(\omega \cdot t + \Phi_{i_0}^{(0)}) + \\ &k \cdot \left( A_{i_0}^{(1)} \cdot \cos(\omega \cdot t + \Phi_{i_0}^{(1)}) + A_{i_0}^{(2)} \cdot \cos(2\omega \cdot t + \Phi_{i_0}^{(2)}) \right). \end{aligned} \quad (10)$$

In this expression, we only have terms sinusoidal with frequency  $f$  and terms sinusoidal with frequency  $2f$ . Thus, in this approximation, the Fourier transform of the acceleration consists of only two components:

- a component corresponding to the main frequency  $f$  (and the corresponding angular frequency  $\omega$ ), and
- a component corresponding to the first harmonic  $2f$ , with the angular frequency  $2\omega$ .

The amplitude  $A_2$  of the first harmonic  $2\omega$  is equal to  $A_2 = k \cdot A_{i_0}^{(2)}$ . The amplitude  $A_1$  of the main frequency  $\omega$  is equal to  $A_1 = A_{i_0}^{(1)} + k \cdot c$  for some constant  $c$  depending on the relation between the phases. Thus, the ratio of these two amplitudes has the form

$$\frac{A_2}{A_1} = \frac{k \cdot A_{i_0}^{(2)}}{A_{i_0}^{(1)} + k \cdot c}. \quad (11)$$

In all the previous formulas, we ignored terms which are quadratic (or of higher order) in terms of  $k$ . If we perform a similar simplification in the formula (11), we conclude that

$$\frac{A_2}{A_1} = k \cdot C, \quad (12)$$

where we denoted  $C \stackrel{\text{def}}{=} \frac{A_{i_0}^{(2)}}{A_{i_0}^{(1)}}$ . In other words, we conclude that the CMV ratio is, in the first approximation, indeed proportional to stiffness.

**Main conclusion.** We have explained why, for reasonable small stiffness levels, we can only see two Fourier components above the noise level: the component corresponding to the vibrating frequency  $f$  and the component corresponding to the first harmonic  $2f$ .

We have also explained the empirical fact that the CMV – the ratio of the amplitudes of the two harmonics – is proportional to the pavement stiffness.

**Case of larger stiffness: analysis and corresponding additional conclusions.** When the stiffness  $k$  is sufficiently large, we can no longer ignore terms which are quadratic or of higher order in terms of  $k$ . In general, the larger the stiffness level, the more terms we need to take into account to get an accurate description of the corresponding dynamics.

Also, when the stiffness  $k$  is small, then, due to the fact that the displacements  $x_i(t)$  are also reasonably small, the products of  $k$  and the terms which are, e.g., cubic in  $x_j(t)$  can be safely ignored. However, when  $k$  is not very small, we need to take these terms into account as well. Using the corresponding expansion of the equations (3), and taking into account more terms in the expansion of  $x_i(k, t)$  in  $k$ , we end up with terms which are cubic (or higher order) in terms of the  $\omega$ -sinusoids  $x_i^{(0)}(t)$ . These terms correspond to triple, quadruple, and higher frequencies  $3f$ ,  $4f$ , etc.

This is exactly what we observe: the higher the stiffness, the more higher order harmonics we see. Thus, this additional empirical fact is also theoretically explained.

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