Derivation of Gross-Pitaevskii Version of Nonlinear Schroedinger Equation from Scale Invariance

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It is known that in the usual 3-D space, the Schroedinger equation
\[ i \cdot \hbar \cdot \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \cdot \nabla^2 \psi + V(x,t) \cdot \psi(x,t) \]
can be derived from scale-invariance.

In view of the fact that, according to modern physics, the actual dimension of proper space may be different from 3, it is desirable to analyze what happens in other spatial dimensions \( D \).

It turns out that while for \( D \geq 3 \) we still get only the Schroedinger’s equation, for \( D = 2 \), we also get the Gross-Pitaevskii version of a nonlinear Schroedinger equation
\[ i \cdot \hbar \cdot \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \cdot \nabla^2 \psi + V(x,t) \cdot \psi(x,t) + c \cdot |\psi|^2 \cdot \psi \]
that describes a quantum system of identical bosons, and for \( D = 1 \), we also get a new nonlinear version of the Schroedinger equation
\[ i \cdot \hbar \cdot \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \cdot \nabla^2 \psi + V(x,t) \cdot \psi(x,t) + \frac{c}{m} \cdot |\psi|^4 \cdot \psi. \]