

Why Linear Interpolation?

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Linear interpolation is the computationally simplest of all possible interpolation techniques. Interestingly, it works reasonably well in many practical situations, even in situations when the corresponding computational models are rather complex. In this talk, we explain this empirical fact by showing that linear interpolation is the only interpolation procedure that satisfies several reasonable properties such as consistency and scale-invariance.

By an *interpolation function*, we mean a function $I(x_1, y_1, x_2, y_2, x)$ which is defined for all $x_1 < x < x_2$ and which has the following properties:

- conservativeness:

$$\min(y_1, y_2) \leq I(x_1, y_1, x_2, y_2, x) \leq \max(y_1, y_2)$$

for all x_i, y_i , and x ;

- x -scale-invariance: $I(a \cdot x_1 + b, y_1, a \cdot x_2 + b, y_2, a \cdot x + b) = I(x_1, y_1, x_2, y_2, x)$ for all $x_i, y_i, x, a > 0$, and b ;
- y -scale invariance: $I(x_1, a \cdot y_1 + b, x_2, a \cdot y_2 + b, x) = a \cdot I(x_1, y_1, x_2, y_2, x) + b$ for all $x_i, y_i, x, a > 0$, and b ;
- consistency:

$$I(x_1, y_1, x_2, y_2, x) = I(x'_1, I(x_1, y_1, x_2, y_2, x'_1), x'_2, I(x_1, y_1, x_2, y_2, x'_2), x)$$

for all x_i, x'_i, y_i , and x ; and

- continuity: the expression $I(x_1, y_1, x_2, y_2, x)$ is a continuous function of x , $I(x_1, y_1, x_2, y_2, x) \rightarrow y_1$ when $x \rightarrow x_1$ and $I(x_1, y_1, x_2, y_2, x) \rightarrow y_2$ when $x \rightarrow x_2$.

Our main result is that the only interpolation function satisfying all these properties is the linear interpolation

$$I(x_1, y_1, x_2, y_2, x) = \frac{x - x_1}{x_2 - x_1} \cdot y_1 + \frac{x_2 - x}{x_2 - x_1} \cdot y_2.$$