What If We Only Know Hurwicz’s Optimism-Pessimism Parameter with Interval Uncertainty?

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Formulation of the problem. In many practical situations, we do not know the exact consequences of each possible action. As a result, instead of single utility value $u$, we can characterize each possible action by the interval $[\underline{u}, \overline{u}]$ of possible utility values. In such situations, decision theory recommends to select an alternative for which the following combination is the largest: $\alpha \cdot \overline{u} + (1 - \alpha) \cdot \underline{u} = \underline{u} + \alpha \cdot (\overline{u} - \underline{u})$, where the parameter $\alpha$—known as Hurwicz’s optimism-pessimism parameter—may be different from different people. The value $\alpha = 1$ corresponds to absolute optimists, the value $\alpha = 0$ describes a complete pessimist, and values between 0 and 1 describe reasonable decision makers.

The parameter $\alpha$ needs to be determined based on a person’s preferences and decisions. Often, however, in different situations, the decisions of the same person correspond to somewhat different values $\alpha$. As a result, instead of a single value $\alpha$, we have the whole range $[\alpha, \overline{\alpha}]$ of possible values. In this case, how should we make decisions?

Our solution. For each $\alpha \in [\alpha, \overline{\alpha}]$, the interval $[\underline{u}, \overline{u}]$ is equivalent to $\underline{u} + \alpha \cdot (\overline{u} - \underline{u})$. This expression is monotonic in $\alpha$, so in general, the original interval $[\underline{u}, \overline{u}]$ is equivalent to the following range of possible values $[\underline{u}_1, \overline{u}_1] = [\underline{u} + \alpha \cdot (\overline{u} - \underline{u}), \underline{u} + \alpha \cdot (\overline{u} - \underline{u})]$. Similarly, the new interval is equivalent to $[\underline{u}_2, \overline{u}_2] = [\underline{u}_1 + \alpha \cdot (\overline{u}_1 - \underline{u}_1), \underline{u}_1 + \alpha \cdot (\overline{u}_1 - \underline{u}_1)]$, etc. At each step, the width of the original intervals decreases by the factor $\alpha$:

$$\underline{u}_k - \overline{u}_k = (\overline{\alpha} - \alpha)^k \cdot (\overline{u} - \underline{u}).$$

In the limit, the intervals $[\underline{u}_k, \overline{u}_k]$ tend to a single point

$$u = \underline{u} + \alpha \cdot (\overline{u} - \underline{u}) + \alpha \cdot (\overline{u} - \underline{u}) \cdot (\overline{\alpha} - \alpha) + \alpha \cdot (\overline{u} - \underline{u}) \cdot (\overline{\alpha} - \alpha)^2 + \ldots$$

The corresponding geometric progression adds to $\underline{u} + \alpha \cdot (\overline{u} - \underline{u})$ for $\alpha = \frac{\alpha}{1 - (\overline{\alpha} - \alpha)}$. This is the desired equivalent value of $\alpha$ for the case when we know $\alpha$ with interval uncertainty—and this is how we should make decisions in this case.