Towards a Natural Interval Interpretation of Pythagorean and Complex Degrees of Confidence

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Formulation of the problem. Often, we only know the expert's degrees of confidence $a, b \in [0,1]$ in statements A and B, and we need to estimate the expert's degree of confidence in $A \,\&\, B$. The algorithm $f_\&(a,b)$ providing the corresponding estimate is known as an *"and"-operation*, or a *t-norm.* One of the most frequently used "and"-operation is $\min(a,b)$. Similarly, one of the most frequently used "or"-operation is $\max(a,b)$.

Often, it is difficult for an expert to describe his/her degree of certainty by a single number a, an expert is more comfortable describing it by range (interval) $[\underline{a}, \overline{a}]$ of possible values. (An alternative way of describing this is as an *intuitionistic fuzzy degree*, i.e., a pair of values $\underline{a}$ and $1-\overline{a}$.) If we know intervals $[\underline{a}, \overline{a}]$ and $[\underline{b}, \overline{b}]$ corresponding to a and b, then the range of possible degree of confidence in $A \,\&\, B$ is formed by values $\min(a,b)$ corresponding to all $a \in [\underline{a}, \overline{a}]$ and $b \in [\underline{b}, \overline{b}]$. Since $\min(a,b)$ is monotonic, this range has the form $[\min(\underline{a}, \underline{b}), \min(\overline{a}, \overline{b})]$. Similarly, the range for $A \vee B$ is $[\max(\underline{a}, \underline{b}), \max(\overline{a}, \overline{b})]$.

A recent paper [1] describes extensions $[\mathrm{absmin}(\underline{a}, \underline{b}), \mathrm{absmin}(\overline{a}, \overline{b})]$ and $[\mathrm{absmax}(\underline{a}, \underline{b}), \mathrm{absmax}(\overline{a}, \overline{b})]$ of the above definitions from $a, b \in [0,1]$ to $a, b \in [-1,1]$, where:

- $\mathrm{absmin}(a,b)$ is a if $|a| < |b|$, b is $|a| > |b|$, and $-|a|$ if $|a| = |b|$ and $a \neq b$,
- $\mathrm{absmax}(a,b)$ is a if $|a| > |b|$, b if $|a| < |b|$, and $|a|$ if $|a| = |b|$ and $a \neq b$.

These operations have nice properties, but what is their meaning?

Our explanation. In addition to closed intervals, let us consider open and semi-open ones. An open end will be then denoted by the negative number: e.g., $(0.3, 0.5]$ is denoted as $[-0.3, 0.5]$, and $(0.3, 0.5)$ as $[-0.3, -0.5]$.

By considering all possible cases, one can show that for two intervals $A = [\underline{a}, \overline{a}]$ and $B = [\underline{B}, \overline{B}]$, the range of possible values $\{\min(a,b) : a \in A, b \in B\}$ is indeed $[\mathrm{abs}\min(\underline{a}, \underline{b}), \mathrm{abs}\min(\overline{a}, \overline{b})]$. Similarly, the range of possible values of $\max(a,b)$ forms the interval $[\mathrm{abs}\max(\underline{a}, \underline{b}), \mathrm{abs}\max(\overline{a}, \overline{b})]$. Thus, we get a natural interval explanation of the extended operations.

[1] S. Dick, R. Yager, and O. Yazdanbakhsh, "On Pythagorean and complex fuzzy set operations", *IEEE Transactions on Fuzzy Systems*, 2016, Vol. 24, No. 5, pp. 1009–1021.