Towards a Natural Interval Interpretation of Pythagorean and Complex Degrees of Confidence

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Formulation of the problem. Often, we only know the expert’s degrees of confidence $a, b \in [0, 1]$ in statements $A$ and $B$, and we need to estimate the expert’s degree of confidence in $A \& B$. The algorithm $f_\land(a, b)$ providing the corresponding estimate is known as an “and”-operation, or a t-norm. One of the most frequently used “and”-operation is $\min(a, b)$. Similarly, one of the most frequently used “or”-operation is $\max(a, b)$.

Often, it is difficult for an expert to describe his/her degree of certainty by a single number $a$, an expert is more comfortable describing it by range (interval) $[a, \pi]$ of possible values. (An alternative way of describing this is as an intuitionistic fuzzy degree, i.e., a pair of values $a$ and $1 - \pi$.) If we know intervals $[a, \pi]$ and $[b, \beta]$ corresponding to $a$ and $b$, then the range of possible degree of confidence in $A \& B$ is formed by values $\min(a, b)$ corresponding to all $a \in [a, \pi]$ and $b \in [b, \beta]$. Since $\min(a, b)$ is monotonic, this range has the form $[\min(a, b), \min(\pi, \beta)]$. Similarly, the range for $A \lor B$ is $[\max(a, b), \max(\pi, \beta)]$.

A recent paper [1] describes extensions $[\min(a, b), \max(\pi, \beta)]$ and $[\min(a, b), \max(\pi, \beta)]$ of the above definitions from $a, b \in [0, 1]$ to $a, b \in [-1, 1]$, where:

- $\min(a, b)$ is $a$ if $|a| < |b|$, $b$ if $|a| > |b|$, and $-|a|$ if $|a| = |b|$ and $a \neq b$,
- $\max(a, b)$ is $a$ if $|a| > |b|$, $b$ if $|a| < |b|$, and $|a|$ if $|a| = |b|$ and $a \neq b$.

These operations have nice properties, but what is their meaning?

Our explanation. In addition to closed intervals, let us consider open and semi-open ones. An open end will be then denoted by the negative number: e.g., $(0.3, 0.5]$ is denoted as $[-0.3, 0.5]$, and $(0.3, 0.5)$ as $[-0.3, -0.5]$.

By considering all possible cases, one can show that for two intervals $A = [a, \pi]$ and $B = [b, \beta]$, the range of possible values $\{\min(a, b) : a \in A, b \in B\}$ is indeed $[\min(a, b), \min(\pi, \beta)]$. Similarly, the range of possible values of $\max(a, b)$ forms the interval $[\max(a, b), \max(\pi, \beta)]$. Thus, we get a natural interval explanation of the extended operations.