Soft Computing Approach to Detecting Discontinuities, on the Example of Detecting Faults in Seismic Analysis

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***Abstract*—Starting from Newton, the main equations of physics are differential equations – which implicitly implies that all the corresponding processes are differentiable – and thus, continuous. However, in practice, we often encounter processes or objects that change abruptly in time or in space. In physics, we have phase transitions when the properties change abruptly. In geosciences, we have sharp boundaries between different layers and discontinuing representing faults. In many such situations, it is important to detect these discontinuities. In some cases, we know the equations, but in many other cases, we do not know the equations, we only know that the corresponding process is discontinuous. In this paper, we show that by applying the soft computing techniques to translate this imprecise knowledge into a precise strategy, we can get an efficient algorithm for detecting discontinuities; its efficiency is shown on the example of detecting a fault based on the seismic signals.**

I. Need to Detect Discontinuities: Formulation of the Problem

Most physical processes are continuous. Starting from Newton, the main equations of physics are differential equations; see, e.g., [4].

This fact implicitly implies that all the corresponding processes are differentiable – and thus, continuous.

Some processes are discontinuous. While most processes are indeed continuous, in practice, we often encounter processes or objects that change abruptly in time or in space. Let us give two simple examples:

- In physics, we have phase transitions when the properties change abruptly.
- In geosciences, we have sharp boundaries between different layers and discontinuing representing faults.

It is often important to detect discontinuities. In many such situations, it is important to detect these discontinuities:

- In civil engineering, discontinuities may indicate crack or faults. Finding them can help us check structural integrity of the corresponding structure – e.g., of the airplane or a spaceship.
- In geosciences, faults are places where most earthquakes originate, so finding the exact locations of faults may help predict where earthquakes can happen in the future.
- In fracking – a method of oil extraction when a liquid is pumped into the oil-containing areas – it is important to detect possible cracks, since through these cracks, chemicals in the pumped liquid can penetrate into the environment (such as aquifers contributing to the drinking waver) where their presence is unwelcome.

Detecting discontinuities is often not easy. In some cases, we know the corresponding equations. In such situations, we can use these equations to develop techniques for detecting discontinuities.

In many other situations, however, we do not know the exact equations describing the process. For example, while we may know that there is a fault, we do not know the exact shape of this fault, and we do not have a good understanding of how this fault interacts, e.g., with seismic waves.

In such situations, all we know is the corresponding processes are discontinuous – but we do not know the equations describing this discontinuity. How can we use this information to detect the discontinuities?

What we do in this paper. At first glance, it looks like the word "discontinuity" has a precise meaning – after all, continuity is a mathematical term, with a precise definition. However, in reality, this mathematical definition is *not* what the geophysicists have in mind when they say that a fault is discontinuous. What they mean is rather a commonsense, informal meaning of this word. What they mean is that at nearby locations, we have similar properties.

Our objective is thus to translate this imprecise meaning into a precise algorithm. In this translation, it is reasonable to use the technique of *fuzzy logic*, techniques specifically designed to transform imprecise ("fuzzy") expert knowledge, knowledge formulated by using words from natural language, into precise computer-understandable terms; see, e.g., [6], [7], [8].

In this paper, we show that by applying these techniques, we can get an efficient algorithm for detecting discontinuities; its efficiency is shown on the example of detecting a fault based on the seismic signals.

II. Soft Computing Approach to Detecting Discontinuities

What does (informally understood) continuity means? From the informal viewpoint, a function $a(t)$ is continuous if small changes in t leads to small changes in a. In other words, if the difference $\Delta t \stackrel{\text{def}}{=} t' - t$ is small, then the difference $\Delta a \stackrel{\text{def}}{=} a(t') - a(t)$ should also be small.

This description is indeed informal. The above description is informal, since it uses an imprecise term "small".

Let use soft computing techniques to describe "small". The main idea of a fuzzy translation of imprecise terms like "small" comes from the fact that for precisely defined properties – e.g., property "less than 10.0" – for each possible value x, the corresponding property is either true or false. In the computer, "true" is usually represented as 1, while "false" is represented by 0. Thus, for a precisely known property, to every possible value x, we assign a value 0 or 1, depending on whether the corresponding property holds or not.

For imprecise properties like "small", the situation is more complicated:

- For small values x, we are absolutely sure that x is small, so it is reasonable to assign 1.
- For large values x, we are sure that these values are not small, so we assign the value 0.
- However, for intermediate values, our understanding is that they are "to some extent" small – something intermediate between "absolutely small" and "absolutely not small".

It is therefore reasonable to describe these intermediate degrees of confidence by using numbers intermediate between the number 1 (corresponding to true) and the number 0 (corresponding to "false").

This, in a nutshell, is the main idea of fuzzy logic: to describe an imprecise property like "small", we assign, to each possible value x of the corresponding property, a number the interval $[0, 1]$ that describes the degree to which, according to the expert, the expert believes that x satisfies this property. This number is usually denoted by $\mu(x)$; the corresponding function is known as the *membership function*.

How can we determine the corresponding degree $\mu(x)$? Well, one way is to explicitly ask the expert to mark his/her degree of confidence that x has a given property (e.g., is small) on a scale from 0 to 1.

What is a reasonable function for small? The larger the positive value x, the less we believe that x is small. Thus, for $x > 0$, the function $\mu(x)$ should be strictly decreasing – until it reaches 0.

Also, intuitively, a negative value $-x$ is small if and only if the corresponding positive value x is small. Thus, we have $\mu(-x) = \mu(x)$, i.e., in other words, $\mu(x) = \mu(|x|)$ for all x.

Different quantities may have different scales. The numerical value of a quantity depends on the measuring unit. For example, if we replace meters with centimeters, all the numerical values of lengths are multiplied by 100: 2 m becomes 200 cm. In general, if we replace the original measuring unit with a new unit which is λ times smaller, then each original value x changes to a new value $x' = \lambda \cdot x$.

In the new units, the mathematical form of the membership function describing the notion "small" will change: for each numerical value x' in the new unit, the corresponding value in the old units is

$$x = \frac{x'}{\lambda}$$

and thus, the corresponding degree of confidence that this value is small is equal to

$$\mu'(x') = \mu(x) = \mu\left(\frac{x'}{\lambda}\right). \tag{1}$$

Resulting relation between the properties "small" corresponding to different quantities. To describe the informal notion of continuity, we need to use the notion "small" for two different quantities:

- the quantity Δt and
- the quantity Δa.

In both cases, the notion of smallness is the same, but the scale may be different. Thus, if we denote by $\mu(\Delta t)$ the membership function corresponding to smallness of Δt, the smallness of Δa can be, in general, be described by the formula (1) for some appropriate re-scaling factor λ:

$$\mu'(\Delta a) = \mu\left(\frac{\Delta a}{\lambda}\right). \tag{2}$$

In these terms, what does continuity mean? We have started with describing continuity as an if-then statement: if Δt is small, then Δa is also small. A statement of the type

"if A then B"

is easy to understand in the case of the traditional 2-valued logic: is simply means that:

- if A is true,
- then B should also be true.

In fuzzy logic, we can add that:

- if A is true with some degree d,
- then B should also be true at least with this degree.

In other words, it means that our degree of belief in B should always be greater than or equal to our degree of belief in A.

In particular, for the if-then statement describing continuity, we conclude that for every Δt, we must have

$$\mu'(\Delta a) \geq \mu(\Delta t).$$

Substituting the expression (2) into this inequality, we conclude that

$$\mu\left(\frac{\Delta a}{\lambda}\right) \geq \mu(\Delta t). \tag{3}$$

Let us simplify this condition. At first glance, the above property sounds rather complicated – and, what is worse, depending on the what exactly membership function we use to describe "small". We will show, however, that the property (3) can be drastically simplified.

First, since $\mu(x) = \mu(|x|)$, we conclude that

$$\mu\left(\frac{|\Delta a|}{\lambda}\right) \geq \mu(|\Delta t|). \quad (4)$$

Now, the membership function is applied only to non-negative values, and we know that for non-negative values, this function is strictly decreasing – until it reaches 0. Thus, for sufficiently small Δt – until we reach 0 – the inequality (4) implies that

$$\frac{|\Delta a|}{\lambda} \leq |\Delta t|,$$

or, equivalently, that

$$\left|\frac{\Delta a}{\Delta t}\right| \leq \lambda. \quad (5)$$

Conclusion. Thus, we can detect discontinuities by comparing the ratio

$$\left|\frac{\Delta a}{\Delta t}\right| = \left|\frac{a(t') - a(t)}{t' - t}\right| \quad (6)$$

with some threshold λ:

- as long as the ratio is below the threshold, we are continuous,
- once the ratio is above the threshold, this is an indication of discontinuity.

Sometimes, this criterion can be simplified even further. In some cases, the values t are equally paced:

$$t_1, \quad t_2 = t_1 + \delta t, \ldots, t_k = t_{k-1} + \delta t, \ldots$$

In such case, the desired ratio (6) is simply proportional to the absolute value of the difference $|a(t_k) - a(t_{k-1})|$. Thus, in this case, we get an even simpler criterion for detecting discontinuity:

- if the difference $|a(t_k) - a(t_{k-1})|$ does not exceed a certain threshold (which is equal to $\lambda \cdot \delta t$), this means that at the location t_k, the process is continuous;
- on the other hand, if the difference

  $$|a(t_k) - a(t_{k-1})|$$

  exceeds this threshold, this means that at this location, there is a discontinuity.

Comment. In this simplified sense, the conclusion seems to be consistent with common sense – which is one more reason to trust it.

III. Let Us Apply The Resulting Discontinuity Criterion to Detecting Faults in Seismic Analysis

Description of the experimental data. In this paper, we used the experimental results from the 2014 Southern California study described in [3]. In this study, more than 1000 seismic sensors were placed on a dense 600 m × 600 m grid on top of one of the known faults – San Jacinto fault; see Fig. 1.

These sensors were in place for a 5-week period. During this period, they recorded many earthquakes, both:

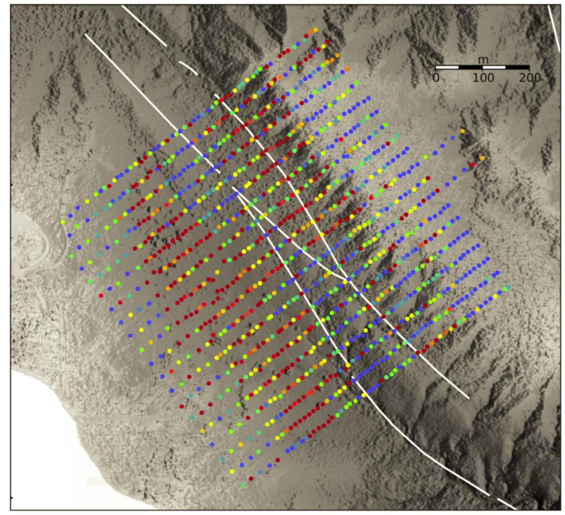

Fig. 1. Sensors placed in the vicinity of San Jacinto fault [3]

- weak earthquakes originating in the vicinity of the fault and
- stronger earthquake that occurred outside the fault.

What we did. For each of the stronger outside earthquakes, for each sensor, we could identify when the signal from this earthquake reached this sensor. For each of the sensors, we then took the recording for a 10-second period period following the arrival of the first (P-wave) signal from the earthquake; see Fig. 2.

Due to the interaction of the signal with the fault and other inhomogeneities, the shape of the signal at different sensors was somewhat different. As a measure of how the earthquake influenced the given sensor, we took the largest possible amplitude a of the seismic signal at this sensor during the selected time interval.

Then, for each straight line of sensors in the direction of wave propagation, and for each sensor $t = 1, 2, \ldots$ in this line, we computed the maximum-amplitude $a(t)$ corresponding to this sensor. Here, the sensors are equally spaced, so we took

$$\delta t = 1.$$

Results. Interestingly, in almost all cases when we selected a line crossing the fault, the difference

$$|a(t) - a(t - \delta t)| = |a(t) - a(t - 1)|$$

spiked when the line of sensor crossed the fault – i.e., when the fault was between the $(k-1)$-st and the k-th sensors. Thus, for each of these lines, we could indeed easily identify the fault as the location at which the difference

$$|a(t) - a(t - 1)|$$

exceeds a certain threshold – in perfect accordance with the above soft-computing-based formula.

Discussion. This accordance with theory was even more amazing to us when we saw that the actual dependence of the amplitude $a(t)$ on t was drastically different for different earthquakes:

- For earthquake waves whose direction was almost orthogonal to the fault, we saw a drastic *increase* in $a(k)$ as the signal crosses the fault.
- In contrast, for earthquake waves whose direction was almost parallel to the fault (and we had an earthquake whose wave direction differed from the fault by only 18°), we saw, vice versa, a sharp *decrease* in the amplitude $a(k)$ followed by sharp increase back to the original amplitude level.

(Our attempts to explain this difference are placed in the appendix.)

In all the cases, however, what was common was the fact that there was a drastic change around the fault – and we can therefore use this change to detect the discontinuities.

IV. Conclusions and Future Work

In this paper, on the example of detecting faults from seismic waves, we have shown that methods based on soft-computing interpretation of discontinuity are very helpful.

We tested this method on the example of detecting the location of the San Jacinto fault, whose location is well known. We hope that this success will enable us also to also detect difficult-to-detect cracks and other faults caused by fracking - and thus, prevent possible ecological disasters.

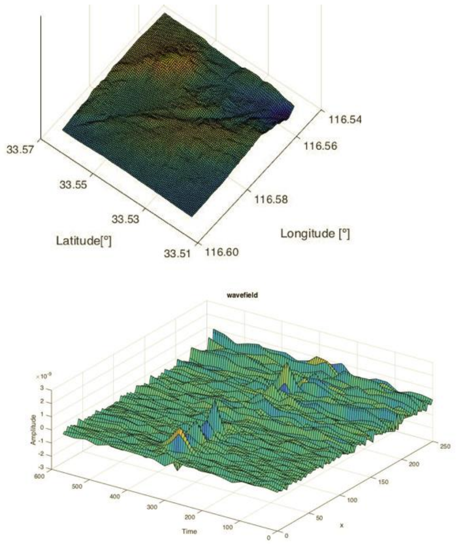

Fig. 2. Wave field for P-wave across the fault

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Appendix: How Can We Explain the Difference Between Behavior of Different Seismic Waves

What we observed: reminder. In our analysis, we observed that when a seismic wave approaches the fault straight ahead – i.e., when its direction is orthogonal to the fault – the amplitude measured by the sensors increases. In contrast, when the earthquake wave approaches the fault at an angle closer to 0, the amplitude measured by the sensor decreases.

Our interpretation. In both cases, when the seismic wave hits the fault, part of its energy is diverted to directions close to orthogonal to the fault. As a result:

- for waves whose direction is almost orthogonal to the fault, we measure a larger amplitude, while
- for waves at a small angle to the fault, the energy decreases.

Such a phenomenon is well known in wave propagation as *scattering*:

- when a wave approaches a point-wise obstacle, the scattered wave goes in all directions around the obstacle;
- when a wave approaches a planar obstacle, we get scattered waves mostly in the directions closed to orthogonal to this obstacle.

Our hope. We hope that the known formulas of scattering seismic waves (see, e.g., [1], [2]) – as well as a similar process of scattering X-rays (see, e.g., [5]) – can help us go from the current idea of detecting the location of the fault to a more detailed description of this fault.