Can Mass Be Negative?

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Abstract

Overcoming the force of gravity is an important part of space travel and a significant obstacle preventing many seemingly reasonable space travel schemes to become practical. Science fiction writers like to imagine materials that may help to make space travel easier. Negative mass – supposedly causing anti-gravity – is one of the popular ideas in this regard. But can mass be negative? In this paper, we show that negative masses are not possible – their existence would enable us to create energy out of nothing, which contradicts to the energy conservation law.

1 Formulation of the Problem

Overcoming the force of gravity is an important part of space travel and a significant obstacle preventing many seemingly reasonable space travel schemes to become practical. Science fiction writers like to imagine materials that may help to make space travel easier. Negative mass – supposedly causing anti-gravity – is one of the popular ideas in this regard. But can mass be negative?

2 Reminder: There Are Different Types of Masses

To properly answer the question of whether negative masses are possible, it is important to take into account that there are, in principle, three types of masses:

- inertial mass m_I that describes how an object reacts to a force F: the object's acceleration a is determined by Newton's law $m_I \cdot a = F$; and
- active and passive gravitational mass m_A and m_P: gravitation force exerted by Object 1 with active mass m_{A1} on Object 2 with passive mass m_{P2} is equal to $F = G \cdot \dfrac{m_{A1} \cdot m_{P2}}{r^2}$, where r is the distance between the two objects; see, e.g., [1, 3].

Can any of these masses be negative?

3 All Three Masses Are Proportional to Each Other

General idea. To answer the above question, let us recall that, due to energy conservation and the properties of anti-particles, all three masses are proportional to each other; see, e.g., [2]. For completeness – and to make sure that the corresponding arguments are applicable to negative masses as well – let us recall the corresponding arguments.

Active and passive masses are proportional to each other: case of positive masses. Let us first show that the active and passive masses are always proportional to each other, i.e., that

$$\frac{m_{A1}}{m_{P1}} = \frac{m_{A2}}{m_{P2}}$$

for every two objects. We will first show it for bodies of positive mass.

Indeed, suppose that for some pair of bodies, this is not true, i.e.,

$$\frac{m_{A1}}{m_{P1}} \neq \frac{m_{A2}}{m_{P2}}.$$

Multiplying both sides of this inequality by both passive masses, we conclude that $m_{A1} \cdot m_{P2} \neq m_{A2} \cdot m_{P1}$. Thus, the gravitational force exerted by Object 1 on Object 2 is different from the gravitational force exerted by Object 2 on Object 1.

So, if we combine these two objects by a rigid rod, the overall force acting on the resulting 2-object system would be different from 0. Thus, if this system was originally immobile, it will start moving with a constant acceleration. We can then stop this system, use the gained kinetic energy to perform some work, and thus, get back to the original configuration – with some work done. We can repeat this procedure as many times as we want. This way, without spending anything, we can get as much work done as we want (and/or as much energy stored as we want). This possibility to get energy from nothing, without changing anything, clearly contradicts to energy conservation law, according to which such perpetuum mobile is impossible.

This contradiction shows that for positive masses, active and passive masses should be proportional to each other: $m_A = \text{const} \cdot m_P$. Thus, if we select the same unit for measuring both active and passive masses, we can conclude that when masses are positive, active and passive masses are equal $m_A = m_P$.

Active and passive masses are proportional to each other: general case. What if at least one of the masses – either active or passive – is negative? In this case, the argument about gaining energy does not necessarily apply: e.g., when the product $m_P \cdot m_A$ is negative, the 2-object system does not gain energy, only loses it.

A slight modification of this thought experiment, however, enables us to gain energy. Indeed, let us consider an object with different active and passive masses $m_A \neq m_P$. Instead of considering this object on its own as before, let us attach is to another object with a big positive mass $M_A = M_P > \max(|m_A|, |m_P|)$. This combination has active mass $C_A = m_A + M_A$ and passive mass $C_P = m_P + M_P$. Since $M_A = M_P > \max(|m_A|, |m_P|)$, both these combined masses are positive. Since $M_A = M_P$ and $m_A \neq m_P$, we conclude that $C_A \neq C_P$. So, the active and passive masses of the combined object are positive and different – and we already know that this leads to a contradiction with the energy conservation law. So, for negative masses, active and passive gravitational masses are also always equal.

Since the active gravitational mass is always equal to the passive gravitational mass, in the following text, we will simply talk about gravitational mass m_G.

Gravitational and inertial masses are proportional to each other: case of positive masses. The important property that will will use is that any type of matter, when combine with the corresponding antimatter, can annihilate, i.e., get transformed into photons, and these photons can get transformed into some other types of matter. For example, we can start with iron and anti-iron, annihilate them, and then get gold and anti-gold. We will also take into account that experiments seems to confirm that matter and corresponding anti-matter have the same inertial and gravitational properties, in particular, the same value of the inertial and gravitational mass; see, e.g., [3].

We want to prove that for all materials, the ratio $\frac{m_G}{m_I}$ of gravitational and inertial masses is the same. Indeed, let us assume that there exist two materials for which this ratio is different, i.e., for which $\frac{m_{G1}}{m_{I1}} \neq \frac{m_{G2}}{m_{I2}}$. Without losing generality, we can assume that the ratio is smaller for the first material: $\frac{m_{G1}}{m_{I1}} < \frac{m_{G2}}{m_{I2}}$. This means that if we select two objects of the same inertial mass $m_{I1} = m_{I2}$ from the first material and from the second material, then the gravitational mass of the first object is smaller: $m_{G1} < m_{G2}$.

We can then get the following scheme for getting energy out of nothing. We place a body and an identical anti-body of the first material at some distance r from the gravitational attractor of some mass M – e.g., from the Earth. We then move both bodies a small distance h away from the Earth. The corresponding force is $F = G \cdot \frac{2m_{G1} \cdot M}{r^2}$, thus the energy that we need to spend for this move is equal to

$$F \cdot h = G \cdot h \cdot \frac{2m_{G1} \cdot M}{r^2}.$$

Once we reached the distance $r + h$, we annihilate both objects, and use the resulting photons to create a pair of a body and anti-body of material 2. Then, we move the new object back to the distance r. This way, the force is equal to $F = G \cdot \frac{2m_{G2} \cdot M}{r^2}$, thus the energy that we gain is equal to

$$F \cdot h = G \cdot h \cdot \frac{2m_{G2} \cdot M}{r^2}.$$

At the distance r, we annihilate both objects, and use the resulting photons to create the original pair of the body and anti-body of Material 1.

Now, we are back to the original state, but, since $m_{G2} > m_{G1}$, we gained more energy that we spent – i.e., as a result, we get energy out of nothing. The impossibility of such a perpetuum mobile shows that, at least for positive masses, gravitational and inertial masses should be proportional to each other: $m_G = \text{const} \cdot m_I$. Thus, if we select the same unit for measuring both gravitational and inertial masses, we can conclude that when masses are positive, gravitational and inertial masses are equal $m_G = m_I$.

Gravitational and inertial masses are proportional to each other: general case. What if at least one of the masses – either gravitational or inertial – is negative? In this case, the above argument does not necessarily apply: e.g., if the inertial mass of some material is negative, we cannot transform it into a material with a positive inertial mass.

A slight modification of this thought experiment, however, enables us to gain energy. Indeed, let us consider an object with different gravitational and inertial masses $m_G \neq m_I$. Instead of considering this object on its own as before, let us attach is to another object with a big positive mass $M_G = M_I > \max(|m_G|, |m_I|)$. This combination has gravitational mass $C_G = m_G + M_G$ and inertial mass $C_I = m_I + M_I$. Since $M_G = M_I > \max(|m_G|, |m_I|)$, both these combined masses are positive. Since $M_G = M_I$ and $m_G \neq m_I$, we conclude that $C_G \neq C_I$. So, the combined object has positive and different gravitational and inertial masses – and we already know that this leads to a contradiction with the energy conservation law. Since the gravitational mass is equal to the inertial mass, in the following text, we will simply talk about the mass m.

Conclusion: unfortunately, there is no such thing as anti-gravity. An unfortunate conclusion is that for every object, whether its mass m is negative or positive, its acceleration in the gravitational field of a body of mass M is determined by the formula $m \cdot a = G \cdot \frac{m \cdot M}{r^2}$, thus $a = G \cdot \frac{M}{r^2}$. This acceleration does not depend on the mass of the attracted body – so all objects follows the same trajectory, negative masses same as positive ones.

4 So Are Negative Masses Possible?

Finally, we can answer the question of whether negative masses are possible. Suppose that negative masses are possible. Then, by attaching an object with a negative mass $m < 0$ to a regular object with a similar positive mass $|m| = -m$, we get a combined object whose overall mass M is 0 (or at least is close to 0). Since the mass M is close to 0, even a very small force F will lead to a huge acceleration $a = \frac{F}{M}$. Thus, without spending practically any energy, we can accelerate the combined object to as high a velocity as we want. Once the object reaches this velocity, we dis-attach the negative-mass object – let it fly away. As a result, we now have an object of positive mass $|m|$ with a very large kinetic energy – and we can use this energy to perform useful work.

This scheme is not as clear-cut as the previous schemes, since here, we do not exactly go back to the original state – we lose a negative-mass body. However, we can do this for negative-mass body of arbitrarily small size – and still gain a lot of energy. Thus, while we *cannot* gain energy and get back to exactly the same original state, we *can* get back to a state which is as close to the original state as we want – and still gain as much energy as we want. This clearly contradicts to the idea of energy conservation. Thus, negative masses are not possible.

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