Why Encubation?

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What is encubation. It is known that some algorithms are feasible, and some
take too long to be practical – e.g., if the running time of an algorithm is $2^n$,
where $n = \text{len}(x)$ is the bit size of the input $x$, then already for $n = 500$, the
computation time exceeds the lifetime of the Universe. In computer science, it
is usually assumed that an algorithm $A$ is feasible if and only if it is polynomial-
time, i.e., if its number of computational steps $t_A(x)$ on any input $x$ is bounded
by a polynomial $P(n)$ of the input length $n = \text{len}(x)$.

An interesting encubation phenomenon is that once we succeed in finding a
polynomial-time algorithm for solving a problem, eventually it turns out to be
possible to further decrease its computation time until we either reach the cubic
time $t_A(x) \approx n^3$ or reach some even faster time $n^\alpha$ for $\alpha < 3$.

How to explain encubation? According to modern physics, the Universe
has $\approx 10^{90}$ particles, and there are $\approx 10^{45}$ moments of time. The number of
moments of time can be obtained if we divide the lifetime of the Universe ($T \approx
20$ billion years) by the smallest possible time: the time that light passes through
the size-wise smallest possible stable particle – a proton.

This means that overall, even if each elementary particle is a processor that
operates as fast as physically possible, the largest possible number of computa-
tional steps that we can perform is $10^{90} \cdot 10^{42} = 10^{132}$. This is the largest
possible number of computational steps $t(n)$.

The largest possible input size comes if you input 1 bit per unit time. Thus,
during the lifetime of the Universe, the largest possible length of the input is
$n \approx 10^{42}$ bits.

If an algorithm is feasible, then for the largest possible length $n$ of the input it
should still perform the physically possible number of steps. For $t(n) \approx n^\alpha$ and
$n \approx 10^{42}$ this means that $t(n) \approx n^\alpha \leq 10^{132}$. Thus, we get $\alpha \leq \frac{132}{42} = \frac{22}{7} \approx 3$ –
which is exactly what we want to explain.

Comment. Since $\frac{22}{7} \approx \pi$, maybe not 3, but $\pi$ is the actual upper bound?