Complexity of computation of the range of a quadratic form and t-norm form on interval domain

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The talk contributes to complexity-theoretic issues in interval computing [4]. Given a quadratic form $q(x) = x^T Qx$ and an interval $\boldsymbol{x}$, it is easy to determine $\min_{x\in\boldsymbol{x}} q(x)$. Convex quadratic programming yields a weakly polynomial algorithm and in some special cases the problem can be solved in strongly polynomial time [1]. Computation of the upper bound $\overline{q} = \max_{x\in\boldsymbol{x}} q(x)$ is NP-hard in general. We show how the complexity of computation of $\overline{q}$ depends on the density of Q [2, 3]. The problem is polynomially solvable for $O(\log n)$ nonzero off-diagonal entries in Q, while it is NP-hard if the number of nonzero entries is of the order n^ε for an arbitrarily small $\varepsilon > 0$.

We also inspect further polynomially solvable cases. We define a sunflower graph on Q and study efficiently solvable cases according to the shape of this graph. Namely, we deal with the case with small sunflower leaves and the case with a restricted number of negative entries in Q.

A natural generalization of a quadratic form is a *t-norm form*, where the cross-terms $x_i x_j$ with $i \neq j$ are replaced by t-norms. Recall that a *t-norm* is a generalization of multiplication (or, from the viewpoint of boolean logic, a generalization of the AND connective). It is a function $t(x_1, x_2)$ satisfying the commutativity and associativity laws, monotonicity in both variables and the axiom that 1 is the identity element. We prove that the computation of $\overline{q}$ remains NP-hard with an arbitrary Lipschitz continuous t-norm.

References

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