real numbers, we mean an algorithm using r_k as an "oracle" (subroutine). This is how computations with real numbers are defined in *computable analysis*.

Once we know the measurement result $\widetilde{x}$ and the upper bound Δ on the measurement error $\Delta x \stackrel{\text{def}}{=} \widetilde{x} - x$, we can conclude that the actual value x belongs to the interval $[\widetilde{x} - \Delta, \widetilde{x} + \Delta]$. In interval analysis, this is all we know: we performed measurements (or estimates), we get intervals, and we want to extract as much information as possible from these results. In particular, we want to know what can we conclude about $y = f(x_1, \ldots, x_n)$, where f is a known algorithm.

In computable (constructive) analysis, we take into account that eventually, we will be able to measure each x_i with higher and higher accuracy. In other words, for each quantity, instead of a *single* interval, we have a *sequence* of narrower and narrower intervals, a sequence that eventually converging to the actual value. From this viewpoint, *Interval analysis is applied constructive analysis* (Yuri Matiyasevich, of 10th Hilbert problem fame).

In this talk, we describe, from this viewpoint, what is a computable set, what is a computable function, and give examples of interval-related positive and negative results of computable analysis.

3.17 Need to Combine Interval and Probabilistic Uncertainty: What Needs to Be Computed, What Can Be Computed, What Can Be Feasibly Computed, and How Physics Can Help

Vladik Kreinovich (University of Texas – El Paso, US)

Need to Combine Interval and Probabilistic Uncertainty: What Needs to Be Computed, What Can Be Computed, What Can Be Feasibly Computed, and How Physics Can Help

In many practical situations, the quantity of interest y is difficult to measure directly. In such situations, to estimate y, we measure easier-to-measure quantities $x_1, \ldots, x_n$ which are related to y by a known relation $y = f(x_1, \ldots, x_n)$, and we use the results $X_1, \ldots, X_n$ of these measurement to estimatey as $Y = f(X_1, \ldots, X_n)$. How accurate is this estimate?

Traditional engineering approach assumes that we know the probability distributions of measurement errors $X_i - x_i$, however, in practice, we often only have partial information about these distributions. In some cases, we only know the upper bounds D_i; in such cases, the only thing we know about the actual value x_i is that it is somewhere in the interval $[X_i - D_i, X_i + D_i]$. Interval computation estimates the range of possible values of y under such interval uncertainty.

In other situations, in addition to the intervals, we also have partial information about the probabilities. In this talk, we describe how to solve this problem in the linearized case, what is computable and what is feasibly computable in the general case, and, somewhat surprisingly, how physics ideas – that initial conditions are not abnormal, that every theory is only approximate – can help with the corresponding computations.

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Report from Dagstuhl Seminar 17481

Reliable Computation and Complexity on the Reals

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Abstract

Naïve computations with real numbers on computers may cause serious errors. In traditional numerical computation these errors are often neglected or, more seriously, not identified. Two approaches attack this problem and investigate its background, Reliable Computing and Computable Analysis.

Methods in Reliable Computing are essentially mathematical theorems, the assumptions of which are verified on the computer. This verification is performed using the very efficient floating point arithmetic. If the verification succeeds, the assertions are true and correct error bounds have been computed; if not, a corresponding message is given. Thus the results are always mathematically correct. A specific advantage of Reliable Computing is that imprecise data are accepted; the challenge is to develop mathematical theorems the assumptions of which can be verified effectively in floating-point and to produce narrow bounds for the solution.

Computable Analysis extends the traditional theory of computability on countable sets to the real numbers and more general spaces by refining continuity to computability. Numerous even basic and simple problems are not computable since they cannot be solved continuously. In many cases computability can be refined to computational complexity which is the time or space a Turing machine needs to compute a result with given precision. By treating precision as a parameter, this goes far beyond the restrictions of double precision arithmetic used in Reliable computing. For practical purposes, however, the asymptotic results from complexity theory must be refined. Software libraries provide efficient implementations for exact real computations.

Both approaches are established theories with numerous important results. However, despite of their obvious close relations these two areas are developing almost independently. For exploring possibilities of closer contact we have invited experts from both areas to this seminar. For improving the mutual understanding some tutorial-like talks have been included in the program. As a result of the seminar it can be stated that interesting joint research is possible.

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