How to Best Process Data If We Have Both Absolute and Relative Measurement Errors: A Pedagogical Comment

Ana Maria Hernandez Posada[1], Maria Isabel Olivarez[1],
Christian Servin[2], and Vladik Kreinovich[1]
[1]Department of Computer Science
University of Texas at El Paso, El Paso, TX 79968, USA
Computer Science and IT Program, El Paso Community College
amhernandezegias@miners.utep.edu, miolivares@miners.utep.edu
cservin1@epcc.edu, vladik@utep.edu

Formulation of the problem. In many practical situations, we need to find the dependence of a quantity y on quantities $x=(x_1,\ldots,x_n)$. Usually, we know the type of the dependence, i.e., we know that $f=f(p,x)$ for some parameters $p=(p_1,\ldots,p_m)$, we just need to find p. For example, the dependence may be linear, then $f(x,p)=\sum\limits_{i=1}^n p_i\cdot x_i+p_{n+1}$.

To find this dependence, we measure x_i and y in several situations k, and find p for which $f(p,x^{(k)})\approx y^{(k)}$ for all k. The measurement error is often caused by a large number of independent factors of about the same size, in which case the Central Limit Theorem implies that it is normally distributed. Usually, it is assumed that the bias is 0, so we only have standard deviation σ.

Sometimes, we have absolute error $\sigma=\text{const}$, in which case we use the usual Least Squares method $\sum\limits_k (y^{(k)}-f(p,x^{(k)}))^2\to\min$. In other cases, we have relative error, in which case we find p for which $\sum\limits_k \dfrac{(y^{(k)}-f(p,x^{(k)}))^2}{(y^{(k)})^2}\to\min$. In practice, however, we usually have both absolute and relative error components: $\Delta y=\Delta y_{\rm abs}+\Delta y_{\rm rel}$, with $\sigma_{\rm abs}=\sigma_0$ and $\sigma_{\rm rel}=\sigma_1\cdot|y|$ for some σ_i. How should we then process data?

Recommendation. In this case, the variance of the measurement error is $\sigma^2=\sigma_0^2+\sigma_1^2\cdot y^2$. So, we use Maximum Likelihood method and maximize the expression

$$\prod_k \frac{1}{\sqrt{2\pi}\cdot\sqrt{\sigma_0^2+\sigma_1^2\cdot(y^{(k)})^2}}\cdot\exp\left(-\frac{(y^{(k)}-f(p,x^{(k)}))^2}{\sigma_0^2+\sigma_1^2\cdot(y^{(k)})^2}\right).$$

In this talk, we present an iterative algorithm for finding p.