Computing with Words – When Results Do Not Depend on the Selection of the Membership Function

Christopher W. Tovar, Carlos Cervantes, Mario Delgado
Stephanie Figueroa, Caleb Gillis, Daniel Gomez
Andres Llausas, Julio C. Lopez Molinar, Mariana Rodriguez
Alexander Wieczkowski, Francisco Zapata, and Vladik Kreinovich
cwtovar@miners.utep.edu, cdcervantes@miners.utep.edu
mdelgado17@miners.utep.edu, sfigueroa2@miners.utep.edu
cbgillis@miners.utep.edu, dagomez8@miners.utep.edu
allausas@miners.utep.edu, jclopezmolinar@miners.utep.edu
mirodriguez9@miners.utep.edu, alwieczkowski@miners.utep.edu
fazg74@gmail.com, vladik@utep.edu

Formulation of the problem. One of the most successful ways to transform natural-language expert knowledge into computer-understandable numerical form is to use fuzzy logic. In fuzzy logic, each imprecise property like “small” is described by a membership function that assigns, to each possible value \( x \), a degree \( \mu(x) \) to which \( x \) is, e.g., small. The problem is that membership functions are subjective. It is therefore desirable to look for cases when the results do not depend on this subjective choice.

Continuity: known example. Intuitively, continuity means that if \( x' \) is close to \( x \), then \( y' = f(x') \) should be close to \( y = f(x) \). In other words, if \( x' - x \) is small, then \( f(x') - f(x) \) should be small. Thus, the degree \( \mu_{\text{small}}^x(f(x') - f(x)) \) cannot be smaller that \( \mu_{\text{small}}^x(x' - x) \). The quantities \( x \) and \( y \) may differ by scale, so \( \mu_{\text{small}}^x(z) = \mu_{\text{small}}^x(K \cdot z) \), thus \( \mu_{\text{small}}^x(K \cdot f(x') - f(x)) \geq \mu_{\text{small}}^x(x' - x) \), hence \( K \cdot |f(x') - f(x)| \leq |x - x'| \) and \( |f(x') - f(x)| \leq |x' - x| \). Thus, the common sense continuity leads to the Lipschitz condition.

New examples. What if we have a relation between \( x \) and \( y \) and not a function? In this case, continuity still implies that \( f(x) \) is a function.

What is the dependence of \( y \) on \( x \) and \( x \) on \( y \)? Then, we have \( |f(x') - f(x)| \leq K^{-1} \cdot |x' - x| \) and \( |x' - x| \leq K \cdot |f(x') - f(x)| \), hence \( |f(x') - f(x)| = K^{-1} \cdot |x' - x| \) for all \( x \) and \( x' \). One can prove that this is only possible when \( f(x) \) is linear.

What if \( f(x) \) is growing? Intuitively, it means that if \( x' \gg x \), then \( f(x') \gg f(x) \). For any membership function for “much larger”, we get \( f(x') - f(x) \geq K \cdot (x' - x) \) for \( x' > x \), i.e., in effect, \( f'(x) \geq K \) for some \( K > 0 \).