Relativistic Effects Can Be Used to Achieve a Universal Square-Root (Or Even Faster) Computation Speedup*

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Abstract. In this paper, we show that special relativity phenomenon can be used to reduce computation time of any algorithm from T to $\sqrt{T}$. For this purpose, we keep computers where they are, but the whole civilization starts moving around the computers – at an increasing speed, reaching speeds close to the speed of light. A similar square-root speedup can be achieved if we place ourselves near a growing black hole. Combining the two schemes can lead to an even faster speedup: from time T to the 4-th order root $\sqrt[4]{T}$.

1 Formulation of the Problem

Need for fast computations. At first glance, the situation with computing speed is very good. The number of computational operations per second has grown exponentially fast, and continues to grow. Faster and faser high performance computers are being designed and built all the time, and the only reason why they are not built even faster is the cost limitations.

However, while, because of this progress, it has indeed become possible to solve many computational problems which were difficult to solve in the past, there are still some challenging practical problems that cannot yet been solved now. An example of such a problem is predicting where a tornado will go in the next 15 minutes. At present, this tornado prediction problem can be solved in a few hours on a high performance computer, but by then, it will be too late. As a result, during the tornado season, broad warning are often so frequent that people often ignore them – and fall victims when the tornado hits their homes. There are many other problems like this.

What can we do – in addition to what is being done. Computer engineers and computer scientists are well aware of the need for faster computations, so computer engineers are working on new hardware that will enable faster computations, and computer scientists are developing new faster algorithms for solving different problems. Some of the hardware efforts are based on the same physical

* This work was supported in part by the US National Science Foundation grant HRD-1242122 (Cyber-ShARE Center of Excellence).

and engineering principles on which the current computers operate, some efforts aim to involve different physical phenomena – such as quantum computing (see, e.g., [5]).

Can we use other physical phenomena as well? We are talking about speeding up computations, i.e., about time, so a natural place to look for such physical phenomena is to look for physical effects that change the rate of different physical processes, i.e., make them run faster or slower.

What we do in this paper. This is what we will do in this paper: we will show how physical phenomena can be used to further speed up computations. Specifically, for this speed-up, in line with the general idea of relativistic computation (see, e.g., [1]), we will be using relativistic effects.

2 Physical Phenomena That Change the Rate of Physical Processes – and How to Use Them to Speed Up Computations

Physical phenomena that change the rate of physical processes: a brief reminder. Unfortunately for computations, there are no physical processes that *speed up* all physical phenomena, but there are two physical processes that *slow down* all physical phenomena; see, e.g., [2], Vol. I, Chapters 15–17, and Vol. II, Chapter 42, and [6], Chapters 2, 24, and 25.

First, according to Special Relativity Theory, if we travel with some speed v, then all the processes slow down. The proper time interval s – i.e., the time interval registered by the observer moving with the speed v – is related to the time interval t measured by the immobile observer by the formula

$$s = t \cdot \sqrt{1 - \frac{v^2}{c^2}}, \tag{1}$$

where c denotes the speed of light. The closer the observer's speed v to the speed of light c, the larger this slow-down.

Second, according to General Relativity Theory, in the gravitational field, time also slows down. For immobile observer in a gravitational field, the proper time interval s is equal to $s = \sqrt{g_{00}} \cdot t$, where t is the time as measured by a distant observer – who is so far away that this observer is not affected by the gravitational field – and g_{00} is the 00-component of the metric tensor g_{ij} that describes the geometry of space-time. In the spherically symmetric (Schwarzschild) solution, we have $g_{00} = 1 - \frac{r_s}{r}$, where r is the distance from the center of the gravitating body and $r_s \stackrel{\text{def}}{=} \frac{2G \cdot M}{c^2}$, where G is the gravitational constant and M is the mass of the central body.

Both slow-down effects have been experimentally confirmed with high accuracy.

How we can use these phenomena to speed up computations. If these phenomena would speed up all the processes, then it would be easy to speed up computations: we would simply place the computers in a body moving with a high speed and/or located in a strong gravitational field, and we would thus get computations faster.

In reality, these phenomena slow down all the processes, not speed them up. So, if we place computers in such a slowed-time environment, we will only slow down the computations. However, we *can* speed up computations if we do the opposite: namely, keep computers in a relatively immobile place with a reasonably low gravitational field, and place our whole civilization in a fast moving body and/or in a strong gravitational field. In this case, in terms of the computers themselves, computations will continue at the same speed as before, but since our time will be slowed down, we will observe much more computational steps in the same interval of proper time (i.e., time as measured by our slowed-down civilization).

In this paper, we analyze what speed up we can obtain in this way – by analyzing the above slowing-down physical phenomena one by one.

3 Possible Special-Relativity Speed-Up: Analysis of the Problem and Resulting Formulas

How to use special relativistic effects for a computational speed-up: reminder. To get a computational speed-up, we can place the computer at the center, and start moving around this computer at a speed close to the speed of light. Since we cannot immediately reach the speed of light or the desired trajectory radius, we need to gradually increase our speed and the radius. Let $v(t)$ denote our speed at time t, and let $R(t)$ denote the radius of our trajectory at moment t.

Preliminary analysis of the problem: simplified computations. According to the above formula (1), a change ds in proper time is related to the change dt in coordinate time (as measured by the computer clock) as $ds = dt \cdot S(t)$, where $S(t) \stackrel{\text{def}}{=} \sqrt{1 - \frac{v^2(t)}{c^2}}$.

The possibility to travel is limited by the need to keep acceleration experienced by all moving persons below or at the usual Earth level g_0. The faster we go, the larger the slow-down effect – and thus, the larger the expected computational speed-up. Thus, to achieve the largest possible computational speed-up, we should accelerate as fast as possible. Since possible accelerations are limited by g_0, this means that, to achieve the largest possible speed-up, we should always accelerate with the maximum possible acceleration g_0.

When a body follows a circular orbit with velocity $v(t)$ and radius $R(t)$, it experiences coordinate acceleration $\frac{d^2x}{dt^2} = \frac{v^2(t)}{R(t)}$. As we accelerate, the velocity gets closer and closer to the speed of light. For large t, the velocity $v(t)$ becomes close to the speed of light $v(t) \approx c$, so we conclude that the following asymptotic equality holds: $\frac{d^2x}{dt^2} \approx \frac{c^2}{R(t)}$. Let us find out what is the value of the experienced acceleration $\frac{d^2x}{ds^2}$.

Here, $\frac{dx}{ds} = \frac{dx}{dt} \cdot \frac{dt}{ds} = \frac{dx}{dt} \cdot \frac{1}{S(t)}$, thus,

$$\frac{d^2x}{ds^2} = \frac{d}{ds}\left(\frac{dx}{dt} \cdot \frac{1}{S(t)}\right) = \frac{dt}{ds} \cdot \frac{d}{dt}\left(\frac{dx}{dt} \cdot \frac{1}{S(t)}\right) = \frac{1}{S(t)} \cdot \frac{d}{dt}\left(\frac{dx}{dt} \cdot \frac{1}{S(t)}\right).$$

When the body follows a circular orbit with a constant speed, the value $S(t)$ is a constant, so we have

$$\frac{d}{dt}\left(\frac{dx}{dt} \cdot \frac{1}{S(t)}\right) = \frac{1}{S(t)} \cdot \frac{d^2x}{dt^2}$$

and thus,

$$\frac{d^2x}{ds^2} = \frac{1}{S^2(t)} \cdot \frac{d^2x}{dt^2} \approx \frac{1}{S^2(t)} \cdot \frac{c^2}{R(t)}.$$

Here, the experienced acceleration $\frac{d^2x}{ds^2}$ should be equal to the usual Earth acceleration g_0, thus

$$g_0 \approx \frac{1}{S^2(t)} \cdot \frac{c^2}{R(t)}.$$

In this case, the relativistic slow-down has the form $S(t) = \frac{c}{\sqrt{g_0 \cdot R(t)}}$. The larger $R(t)$, the larger the slow-down effect and thus, the larger the expected computational speed up. All the speeds are limited by the speed of light, thus, we have $R(t) \le v_0 \cdot t$, where $v_0 < c$ is the speed with which we increase the radius. To increase the computational speed-up effect, let us consider the case when $R(t) = v_0 \cdot t$. In this case, the relativistic slow-down effect has the form

$$S(t) \approx C^{-1} \cdot t^{-1/2},$$

where we denoted $C \stackrel{\text{def}}{=} \frac{\sqrt{g_0 \cdot v_0}}{c}$.

From $S(t) = \sqrt{1 - \frac{v^2(t)}{c^2}} = \frac{c}{\sqrt{g_0 \cdot v_0 \cdot t}}$, we conclude that $1 - \frac{v^2(t)}{c^2} = \frac{c^2}{g_0 \cdot v_0 \cdot t}$, and thus,

$$v(t) = c \cdot \sqrt{1 - \frac{c^2}{g_0 \cdot v_0 \cdot t}}.$$

At any moment of time t, we get the following relation between the increase dt in corresponding time and the increase ds in proper time (i.e., time experienced by us):

$$\frac{ds}{dt} = S(t) \approx C^{-1} \cdot t^{-1/2},$$

hence $ds \approx C^{-1} \cdot dt \cdot t^{-1/2}$. Integrating both sides of this asymptotic equality, we conclude that $s \approx 2C^{-1} \cdot \sqrt{t}$.

Once the computers finish their computations at time T, they need to send us the results. This can be done with the speed of light. So, at each moment $t \geq T$ of coordinate time, the signal reaches the distance $c \cdot (t - T)$ from the computers' location. We receive this signal when it reaches our location, i.e., at a moment t_r for which $c \cdot (t_r - T) = R(t_r) = v_0 \cdot t_r$. So, $(c - v_0) \cdot t_r = c \cdot T$ and $t_r = \dfrac{c}{c - v_0} \cdot T$. At this moment, our experienced time s_r is equal to

$$s_r \approx 2C^{-1} \cdot \sqrt{t_r} = 2C^{-1} \cdot \sqrt{\frac{c}{c - v_0}} \cdot \sqrt{T}.$$

Thus, in comparison with the usual (stationary) computations which would require time T, we indeed get a square-root computational speed-up.

This is probably all we can get. Please note that this square root speedup is probably all we can gain: indeed, we tried to extract as much slowing down as possible with the limitation that the acceleration does not exceed g_0. A further relativistic slow-down would probably require having accelerations much higher than our usual level g_0.

Detailed analysis and the resulting computational speed-up scheme. In the above simplified computations, we used the formulas which are valid for the case when the body is moving with a constant speed along the same circular orbit. In our scheme, both the speed and the radius $R(t)$ increase with time. Let us now perform a more accurate analysis, that takes these changes into account and leads to the same asymptotic speed-up. To be more precise, we will show that it is possible, for each $\varepsilon > 0$, to achieve a speed-up from T to $T^{1/2+\varepsilon}$. Since this value ε can be arbitrarily small, from the practical viewpoint, this means that, in effect, we get the square root speed-up.

To speed up computations, we place computers where they are now, and start moving the whole civilization. All the motion will be in a plane, with the civilization following – after some preparation time t_0 – a logarithmic spiral trajectory, i.e., a trajectory that in polar coordinates (R, φ) takes the form $R = R_0 \cdot \exp(k \cdot \varphi)$, i.e., equivalently, $\varphi = K \cdot \ln(R/R_0) = K \cdot \ln(R) - K \cdot \ln(R_0)$, where we denoted $K \stackrel{\text{def}}{=} k^{-1}$. To show that the corresponding speedup can be achieved, we will take $K = \dfrac{v_0/c}{\sqrt{1 - \dfrac{v_0^2}{c^2}}}$.

For the dependence of the distance $R(t)$ on time t, we consider the following formula

$$R(t) = \sqrt{c^2 - v_0^2} \cdot t - c_0 \cdot t^{2\varepsilon},$$

for an appropriate constant c_0 (that will be determined later). We will show that for an appropriately selected value c_0, the perceived acceleration $a \stackrel{\text{def}}{=} \left\| \dfrac{d^2 x_i}{ds^2} \right\|$

will not exceed the Earth's level g_0, and that this trajectory will indeed lead to the $T \to T^{1/2+\varepsilon}$ speedup. Since we are considering only moments after some time t_0, it is sufficient to prove that the asymptotic expression for the acceleration a does not exceed $g_0' \stackrel{\rm def}{=} g_0 - \delta$ for some small $\delta > 0$; this will guarantee that the acceleration is smaller than g_0 for all moments t starting with some moment t_0.

Let us first estimate the relativistic slow-down. In the usual Cartesian coordinates, the trajectory has the form

$$x(t) = R(t) \cdot \cos(K \cdot \ln(R) - K \cdot \ln(R_0)), \quad y(t) = R(t) \cdot \sin(K \cdot \ln(R) - K \cdot \ln(R_0)).$$

Differentiating these formulas with respect to coordinate time t, we conclude that

$$\frac{dx}{dt} = R'(t) \cdot \cos(K \cdot \ln(R) - K \ln(R_0)) - R(t) \cdot \sin(K \cdot \ln(R) - K \cdot \ln(R_0)) \cdot \frac{K}{R(t)} \cdot R'(t) =$$

$$R'(t) \cdot (\cos(K \cdot \ln(R) - K \ln(R_0)) - K \cdot \sin(K \cdot \ln(R) - K \ln(R_0))),$$

where $R'(t)$ denotes the derivative of the function $R(t)$, and similarly

$$\frac{dy}{dt} = R'(t) \cdot (\sin(K \cdot \ln(R) - K \cdot \ln(R_0)) + K \cdot \cos(K \cdot \ln(R) - K \cdot \ln(R_0))).$$

Substituting these expressions into the formula

$$v^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2,$$

taking into account that terms proportional to the product of sine and cosine cancel each other and that $\sin^2(z) + \cos^2(z) = 1$, we conclude that

$$v^2 = (R'(t))^2 \cdot (1 + K^2).$$

From the formula for $R(t)$, we get $R'(t) = \sqrt{c^2 - v_0^2} - c_0 \cdot 2\varepsilon \cdot t^{-(1-2\varepsilon)}$, thus

$$(R'(t))^2 = c^2 - v_0^2 - 2\sqrt{c^2 - v_0^2} \cdot c_0 \cdot 2\varepsilon \cdot t^{-(1-2\varepsilon)} + o,$$

where o denotes terms that are asymptotically smaller than all the terms present in this formula. So, $v^2 = (c^2 - v_0^2) \cdot (1 + K^2) - 2\sqrt{c^2 - v_0^2} \cdot c_0 \cdot 2\varepsilon \cdot (1 + K^2) \cdot t^{-(1-2\varepsilon)} + o$. By our selection of K, the first term in the formula for v^2 is equal to c^2, so $v^2 = c^2 - c_1 \cdot t^{-(1-2\varepsilon)} + o$, where we denoted $c_1 \stackrel{\rm def}{=} 2\sqrt{c^2 - v_0^2} \cdot c_0 \cdot 2\varepsilon \cdot (1 + K^2)$. Thus, $1 - \dfrac{v^2}{c^2} = c_2 \cdot t^{-(1-2\varepsilon)} + o$, where $c_2 \stackrel{\rm def}{=} \dfrac{c_1}{c^2}$ and hence, the relativistic slow-down is equal to $S(t) = c_3 \cdot t^{-(1/2-\varepsilon)} + o$, where $c_3 \stackrel{\rm def}{=} \sqrt{c_2}$. So, asymptotically, $\dfrac{dt}{ds} = \dfrac{1}{S(t)} \sim t^{1/2-\varepsilon}$ and $\left(\dfrac{dt}{ds}\right)^2 \sim t^{1-2\varepsilon}$.

The perceived acceleration has the form $a = \|a_i\|$, where

$$a_i = \frac{d}{ds}\left(\frac{dx_i}{ds}\right) = \frac{d}{ds}\left(\frac{dx_i}{dt} \cdot \frac{dt}{ds}\right) = \frac{dt}{ds} \cdot \frac{d}{dt}\left(\frac{dx_i}{dt} \cdot \frac{dt}{ds}\right) =$$

$$\left(\frac{dt}{ds}\right)^2 \cdot \frac{d^2x_i}{dt^2} + \frac{dt}{ds} \cdot \frac{dx_i}{dt} \cdot \frac{d}{dt}\left(\frac{dt}{ds}\right).$$

In the second term in the expression for a_i, we have $\frac{dt}{ds} \sim t^{1/2-\varepsilon}$, $\frac{dx_i}{dt} \sim \text{const}$, and $\frac{d}{dt}\left(\frac{dt}{ds}\right) \sim t^{-(1/2+\varepsilon)}$, so the product of these three factors is $\sim t^{-2\varepsilon}$ and thus, tends to 0 as t increases.

In the first term in the expression for a_i, from the above formula for $\frac{dx}{dt}$, we get

$$\frac{d^2x}{dt^2} = R'' \cdot (\cos(.) - K \cdot \sin(.)) - R' \cdot (\sin(.) + K \cdot \cos(.)) \cdot \frac{K}{R} \cdot R'.$$

For the first term in this expression, we have $R'' \sim t^{-(2-2\varepsilon)}$, so due to the above asymptotic for the factor $\left(\frac{dt}{ds}\right)^2$, the product of term proportional to R'' and this factor is $\sim t^{-1}$ – and thus, also tends to 0 as t increases.

For the remaining term, since $R' \sim t^{-(1-\varepsilon)}$ and $R \sim t$, the term proportional to $R' \cdot \frac{K}{R} \cdot R'$ is $\sim t^{-(1-2\varepsilon)}$ and thus, the product of this term and the factor $\left(\frac{dt}{ds}\right)^2 \sim t^{1-2\varepsilon}$ is asymptotically a constant – and a constant proportional to c_0.

A similar conclusion can be made about $\frac{d^2y}{dt^2}$. So, overall, a_i is bounded by a constant proportional to c_0. Hence, by appropriately selecting c_0, we can make this term – and thus, the whole expression a_i – as small as needed, in particular, smaller than the desired acceleration bound g_0'.

Let us now show that in this scheme, we indeed get the desired speed-up. Indeed, here, $\frac{ds}{dt} \sim t^{-(1/2-\varepsilon)}$, so for the proper time $s = \int \frac{ds}{dt}\, dt$ we get $s(t) \sim t^{1/2+\varepsilon}$.

Suppose that the centrally located computer finishes its computations at time T, and immediately sends the result to us. This result travels to us with the speed of light c. Let t_r denote the moment of (coordinate) time at which we receive this result. At this moment of time, we are at the distance $R(t_r)$, so it took the signal time $\frac{R(t_r)}{c}$ to reach us. Thus, $T + \frac{R(t_r)}{c} = t_r$. Asymptotically, $R(t) \sim \sqrt{c^2 - v_0^2} \cdot t$, so for large T, the above formula takes the following asymptotic form $T + \frac{\sqrt{c^2 - v_0^2}}{c} \cdot t_r = t_r$, thus $T = t_r \cdot \left(1 - \sqrt{1 - \frac{v_0^2}{c^2}}\right)$ and $t_r = \frac{T}{1 - \sqrt{1 - \frac{v_0^2}{c^2}}} \sim T$.

We have shown that our proper (perceived) time s depends on the coordinate time t as $s(t) \sim t^{1/2+\varepsilon}$. Thus, by our clocks, we get the result of the computation at the moment of time $s(t_r) \sim T^{1/2+\varepsilon}$. So, we indeed get a square root speed-up.

How realistic is this scheme? How big a radius do we need to reach a reasonable speedup? As we will show, the corresponding radius is – by astronomical standards – quite reasonable. Indeed, for large t, when $v \approx c$, the above formulas relating $S(t)$ and $R(t)$ leads to

$$R(t) \approx \frac{c^2}{g_0} \approx \frac{(3 \cdot 10^8 \text{ m/sec})^2}{10 \text{ m/sec}^2} = 9 \cdot 10^{15} \text{ m}.$$

This radius can be compared with a light year – the distance that the light travels in 1 year – which is equal to

$$\approx (3 \cdot 10^8 \text{ m/sec}) \cdot (3 \cdot 10^7 \text{ sec/year}) \cdot (1 \text{ year}) = 9 \cdot 10^{15} \text{ m},$$

so for $v(t) \approx c$, the radius should be about 1 light year.

With time t, the radius is proportional to t, and the computational speed-up is proportional to $\sqrt{t}$. Thus, the radius grows as the square of the computational speed-up. So:

- to get an order of magnitude (10 times) speedup, we need an orbit of radius $10^2 = 100$ light years – reaching to the nearest stars;
- to get a two orders of magnitude (100 times) speedup, we need an orbit of radius $100^2 = 10^4$ light years – almost bringing us to the edge of our Milky Way Galaxy;
- to get a three orders of magnitude (1000 times) speedup, we need an orbit of radius $1000^2 = 10^6$ light years;
- with an orbit of the same radius as the radius of the Universe $R(t) \approx 20$ billion $= 2 \cdot 10^{10}$ light years, we can get $\sqrt{2 \cdot 10^{10}} \approx 1.5 \cdot 10^5$ speedup – more than hundred thousand times speedup.

This is similar to a quantum speedup. The above square root speedup is similar to the speedup of Grover's quantum algorithm for search in an unsorted array [3–5]; the difference is that:

- in quantum computing, the speedup is limited to search in an unsorted array, while
- in the above special-relativity scheme, we get the same speedup for *all* possible computations.

Comment. In Russia – where we are from – to ring the church bell, the bell-ringer moves the bell's "tongue" (clapper). In Western Europe, they move the bell itself. This example is often used in Russian papers on algorithm efficiency, with an emphasis on the fact that, in principle, it is possible to use a third way to ring the bell: by shaking the whole bell tower. In these papers, this third way is mentioned simply as a joke, but, as the above computations show, this is exactly what we are proposing here: since we cannot reach a speedup by making the computer move, we instead leave the computers intact and move the whole civilization.

Speculation. How can we check whether an advanced civilization is already using this scheme? In this scheme, a civilization rotates around a center, increasing its radius as it goes – i.e., follows a spiral trajectory. In this process, in order to remain accelerating, the civilization needs to gain more and more kinetic energy. The only way to get this energy is to burn all the burnable matter that it encounters along its trajectory. As a result, along the trajectory, where the matter has been burned, we have low-density areas.

Thus, as a trace of such a civilization, we are left with a shape in which there are spiral-shaped low-density areas starting from some central point. But this is exactly how our Galaxy – and many other spiral galaxies – look like. So maybe this is how spiral galaxies acquired their current shape?

4 Possible General-Relativity Speed-Up: Analysis of the Problem and Resulting Formulas

Idea. If we keep the computers where they are now, and place the whole civilization (but *not* the computers) in a strong gravitational field, by moving the civilization close to a far away massive body, then our proper time will slow down. Thus, the computations that take the same coordinate time t will require, in terms of our proper time s, much fewer seconds.

Analysis of the problem. According to the Schwarzschild's formula for the gravitational field of a symmetric body of mass $M(t)$ at a distance $R(t)$ from the center, the change in the proper time ds (as experience by this body) is related to the change dt in time t as measured by the distant observer by the formula $ds = \varepsilon(t) \cdot dt$, where $\varepsilon(t) \stackrel{\text{def}}{=} \sqrt{1 - \frac{r_s}{R(t)}}$ and the parameter r_s (known as the *Schwarzschild radius*) is equal to

$$r_s \stackrel{\text{def}}{=} \frac{2G \cdot M(t)}{c^2};$$

see, e.g., [2], Vol. II, Chapter 42, and [6], Chapters 24 and 25.

We want to have as large computational speed-up as possible, so we need to make sure that the corresponding slow-down is as drastic as possible, i.e., that the slow-down factor $\varepsilon(t)$ is as small as possible. For a given r_s this means that we should take $R(t)$ to be as small as possible – i.e., we want to be able to get as close to the Schwarzschild radius as possible. For usual celestial bodies, the radius r_s is well within them: e.g., for our Sun, this radius is equal to 3 km, much smaller than the Sun's size of 1 million km. The only bodies for which their size is smaller than the Schwarzschild radius are black holes. Thus, in this scheme, the civilization should move close to a black hole.

Getting too close to the black hole is dangerous: if we get to the surface $R = r_s$ (known as the *event horizon*), we will never be able to get back to our world or even send a signal back to our world. Thus, it is desirable to always keep ourselves at a certain safe distance d_0 from the event horizon, a safe distance that enables us to move back if some unexpected fluctuation brings us too close to it. So, the closest we can get to the black hole is at the distance $R(t) = r_s + d_0$, for which $r_s = R(t) - d_0$. For these values, the slow-down factor takes the form

$$\varepsilon(t) = \sqrt{1 - \frac{R(t) - d_0}{R(t)}} = \sqrt{\frac{d_0}{R(t)}}.$$

Thus, to decrease this factor – and thus, to get larger and larger computational speed-up – we need to increase $R(t)$. Since we want to keep r_s to be equal to $R(t) - d_0$, this means that we need to also increase r_s – and since r_s is proportional to the mass $M(t)$ of the black hole, this means that we have to continuously increase its mass.

How fast can we increase the radius? Probably we cannot grow $R(t)$ faster than the speed of light – since otherwise, in the coordinates of the distant observer, we will have a physically impossible faster-than-light process. So, the fastest we can grow is at some speed v_0 not exceeding the speed of light. In this case, $R(t) = v_0 \cdot t$, so the speed-up is proportional to $\varepsilon(t) \sim t^{-1/2}$, and, similarly to the special relativity case, we get a square-root computational speed-up.

Resulting speedup scheme. To speed up computations, we place computers where they are now. Then we look for a faraway massive black hole, so far away that its gravitational effect on the computers is negligible.

Then we ourselves move close to this black hole, so that our distance from this black hole changes with time t as $R(t) = v_0 \cdot t$. While we are doing that, we are increasing the black hole's mass, so that its mass at time t becomes equal to $M(t) = \dfrac{c^2 \cdot (R(t) + d_0)}{2G}$, where G is the gravitational constant.

Once the computers finish their computations, they send the results to us by a direct light-speed signal.

In this scheme, we also get a square-root speedup.

This is probably all we can get. Please note that, similarly to the special relativity scheme, this square root speedup is probably all we can gain: indeed, we tried to extract as much slowing down as possible. A further speedup would probably bring too dangerously close to the event horizon.

5 Ideally, We Should Use Both Speedups

Moving at a speed close to the speed of light decreases the proper time from the original value t to a much smaller amount $s \sim \sqrt{t}$. Similarly, a location near a black hole also decreases the observable computation time to a square root of its original value.

Thus, if we combine these two schemes – i.e., place ourselves near an ever-increasing black hole and move (together with this black hole) at a speed close to the speed of light, we will get both speedups, i.e., we will replace the perceived computation time from T to $\sqrt{\sqrt{T}} = \sqrt[4]{T}$.

Acknowledgments

The authors are greatly thankful to the anonymous referees for valuable suggestions.

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