Interval Computations as Applied Constructive Mathematics: from Shanin to Wiener and Beyond

Vladik Kreinovich
Department of Computer Science
University of Texas at El Paso
El Paso, Texas 79968, USA, vladik@utep.edu

Many algorithms of constructive mathematics are unfeasible, exhaustive-search algorithms. This fact makes many results of constructive mathematics - originally invented to make mathematical results computable and thus more practically useful - too far from practical applications.

How can we make constructive mathematics more realistic? Yuri Matiyasevich, renowned for his solution to the 10th Hilbert's problem, observed that while algorithms of constructive mathematics assume that we have inputs known with increasing accuracy, in practice, the accuracy is fixed. At any given moment of time, we only have a single measurement result $\tilde{x}$, corresponding to the currently available accuracy Δ; see, e.g., [8]. As a result, the only information that we have about the (unknown) actual value x of the measured quantity is that it belongs to the interval $\mathbf{x} = [\tilde{x} - \Delta, \tilde{x} + \Delta]$. Given a data processing algorithm $y = f(x_1, \ldots, x_n)$ and intervals $\mathbf{x}_1, \ldots, \mathbf{x}_n$ corresponding to the inputs, we must therefore describe the corresponding range of possible values of y. This problem is called the problem of *interval computations*, or *interval analysis*.

In this problem, techniques borrowed from constructive mathematics work so well that many researchers – including Yu. V. Matiyasevich himself – consider interval analysis Applied Constructive Mathematics. Interval analysis has numerous practical applications ranging from robotics to planning spaceship trajectories to chemical engineering; see, e.g., [1, 2, 3, 4, 5, 6, 7].

The main idea of interval computations can be traced to Norbert Wiener [9, 10]. Its algorithms were developed by Ramon Moore in the late 1950s and early 1960s. (And, by the way, Yuri Matiyasevich boosted this area by organizing conferences and by helping to launch a journal – then called *Interval Computations* – which remains, under the new, somewhat more general title *Reliable Computing*, the main journal of the interval computations community.)

In this talk, we will expand on this relation, and provide a brief overview of the main achievements of interval computations – and of their relation to constructive mathematics.

References

[1] V. G. Dmitriev, N. A. Zheludeva, and V. Kreinovich, *Applications of interval analysis methods to estimate algorithms errors in measuring systems*, Measurement, Control, Automatization No. 1(53) (1985), 31–40 (in Russian).

[2] Interval Computations website http://www.cs.utep.edu/interval-comp

[3] L. Jaulin, M. Kieffer, O. Didrit, and E. Walter, *Applied Interval Analysis, with Examples in Parameter and State Estimation, Robust Control and Robotics*, Springer-Verlag, London, 2001.

[4] R. B. Kearfott, *Rigorous Global Search: Continuous Problems*, Kluwer, Dordrecht, 1996.

[5] R. B. Kearfott and V. Kreinovich (eds.), *Applications of Interval Computations*, Kluwer, Dordrecht, 1996.

[6] V. Kreinovich, A. Lakeyev, J. Rohn, and P. Kahl, *Computational complexity and feasibility of data processing and interval computations*, Kluwer, Dordrecht, 1998.

[7] R. E. Moore, R. B. Kearfott, and M. J. Cloud, *Introduction to Interval Analysis*. SIAM Press, Philadelphia, Pennsylviania, 2009.

[8] S. G. Rabinovich, *Measurement Errors and Uncertainty: Theory and Practice*, Springer Verlag, New York, 2005.

[9] N. Wiener, *A contribution to the theory of relative position*, Proc. Cambridge Philos. Soc. **17** (1914), 441–449.

[10] N. Wiener, *A new theory of measurement: a study in the logic of mathematics*, Proceedings of the London Mathematical Society **19** (1921), 181–205.