VII Международная научная конференция
«Математическое и компьютерное моделирование»
Омск, ОмГУ, 22 ноября 2019 г.

УДК

E. Nevarez, J. R. Caylor, J. L. Faith, I. I. Martinez Huerta,
O. Kosheleva, and V. Krenovich
University of Texas at El Paso,
El Paso, Texas, USA
enevarez1@miners.utep.edu, jrcaylor@miners.utep.edu,
jlfaith@miners.utep.edu, iimartinezh@miners.utep.edu,
olgak@utep.edu, vladik@utep.edu

WHY WE MOSTLY USE 2-, 3- AND 5-BASED NUMBER SYSTEMS?

What number systems do we use? Officially, we only use the decimal system, with base 10 = 2 * 5, but in practice, when we count, we also use dozens 12 = 2 * 2 * 3, half-dozens 6 = 2 * 3, etc.

Languages show us that in the past, some of used other bases. For example, in French and in Spanish, 20 is described by a different word than all other multiples of 10, which shows that in the past, people used 20 = 2 * 2 * 5 as the base, In Russian, 40 is described by a different word "sorok" – there is even an expression "sorok sorokov" (40 of 40s) for 40 * 40, which shows that the number 40 = 2 * 2 * 2 * 5 was indeed used as a number base.

Historical documents show other number bases: Mayan used base 20, Babylonians used base 60 = 2 * 2 * 3 * 5, etc.; see, e.g., [1, 2] and references therein.

In all these cases, we use numbers formed by multiplying the first three prime numbers: 2, 3, and 5. Why?

Why not 7? We use 7 often: e.g., we combine days into 7-day weeks, but there does not seem to be a widely spread tradition of using base-7 numbers for computing. There is even less evidence of using 11, 13, and larger prime numbers. How can we explain this?

One possible explanation comes from the need to consider areas and volumes. When we measure areas – e.g., when buying and selling land – then for each base b, in addition to the original unit, we have a b^2 times lager unit. For example, in the US system, 1 yard is equal to 3 feet. If we want to measure distance and the foot is too small a unit, we can use yards. Similarly, if we measure area and the square foot is too small a unit, we can use square yards, and a square yard is equal to 3^2 square feet.

Similarly, when we measure volumes – e.g., when buying or selling wine or olive oil -- then with each original unit of volume, we get a new unit which is b^3 times larger. For example, a cubic yard is equal to 3^3 cubic feet.

Sometimes, we buy area-related things and sell volume-related things in return. For example, a farmer may want to sell his olive oil crop and use this money to buy some extra land. In such exchanges, it would be convenient to make sure that the cube of the corresponding base is either equal to the exact square of some number or, if this is not possible, at least be close to some square, so that the negotiations can succeed with one side paying a small difference of 1 or 2 units.

In precise terms, we look for numbers b for which b^3 is close to some value v^2, i.e., for which the absolute value $|b^3 – v^2|$ of the corresponding difference does not exceed 2.

The cases when this difference is 0, i.e., when $b^3 = v^2$, are easy to describe: these are the cases when for some integer t, we have $b = t^2$ and $v = t^3$. For example, we can take t = 2, then b = 4 and v = 8. We can take t = 3, then b = 9 and v = 27. In all these cases, we have numbers formed from 2, 3, and 5. To use another prime number – the smallest of which is 7 – we need $v = 7^3 = 343$, too large a number to serve as a base for a number system.

To find all the cases when the difference is plus minus 1 or plus minus 2, we used a simple program to check all the pairs (b, v). To be on the safe side, we tested all the pairs for which both b and v do not exceed 10 000. Interestingly, among such pairs, only for two pairs the absolute value of the difference does not exceed 2: namely, we have $3^2 – 2^3 = 9 – 8 = 1$ and $3^3 – 5^2 = 27 – 25 = 2$.

Thus, from this viewpoint, reasonable bases are 2, 3, and 5 – which explains why such bases are mostly used. This also explains why, in spite of the prevalence of the decimal system that only uses 2 and 5, we also continue to count in dozens and half-dozens (that use 3): the closest values to 2^3 and 5^2 are powers of 3.

References

1. *Merzbach U.C., Boyer C.B.* A history of mathematics.—Hoboken, New Jersey: John Wiley & Sons, 2011.
2. Kosheleva O., *Villaverde K.* How interval and fuzzy techniques can improve teaching.—Cham, Switzerland: Springer, 2018.