

How to Assign Grades to Tasks so as to Maximize Student Efforts

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Abstract

In some classes, students want to get a desired passing grade (e.g., C or B) by spending the smallest amount of effort. In such situations, it is reasonable for the instructor to assign the grades for different tasks in such a way that the resulting overall student's effort is the largest possible. In this paper, we show that to achieve this goal, we need to assign, to each task, the number of points proportional to the efforts needed for this task.

1 Formulation of the Problem

In some cases, students try to minimize their efforts. In the ideal world, students should apply the maximal effort when studying for all their classes. In reality, students usually have a limited amount of time. As a result, while they concentrate their efforts on their major classes, they limit their efforts in other classes to a necessary minimum – usually, the minimum effort needed to get a passing grade in this class.

This phenomenon is especially frequent when students take requires classes outside their major discipline – e.g., when engineering students take required humanity classes or when biology majors take math and/or computing classes which are not directly related to their discipline.

How can instructors increase the students' efforts in these classes. For classes in which students minimize their efforts, instructors try to maximize the student efforts – to make sure that even with the current attitude, the students learn as much of the topic as possible. Since all these students care about is their overall grade for this class, the only thing that the instructor controls is which proportion of the grade goes for each task. How can we assign these grades so as to maximize the student efforts?

Towards a precise formulation of the problem. The overall grade for the classes is usually computed as a weighted average of grades for different tasks, i.e., in effect, as the sum of partial grades given for each task. The ideal case – usually described by $I = 100$ points – corresponds to the case when the student gets the maximum possible number of points for each of the tasks.

Let n denote the total number of tasks, and let m_i denote the maximum number of points that a student can get for each task. Then, we have $I = \sum_{i=1}^n m_i$.

Let e_i denote the amount of effort (e.g., measured by the time of intensive study) that a student needs to get the maximum number of point m_i in the i -th task, and let $E = \sum_{i=1}^n e_i$ denote the overall effort needed to get a perfect grade m_i on all the tasks – and thus, the perfect grade I for the class.

As we have mentioned, the students do not always apply the maximum effort in studying. Let a_i be the actual effort that the student applies to the i -th task – e.g., into studying the i -th part of the material. (A student may be studying more than needed, but we only count the time that the student studies for the corresponding task.) Since the effort e_i already provides a perfect mastery of the i -th task, we assume that $a_i \leq e_i$.

In the first approximation, it is reasonable to assume that the number of points gained by the student is proportional to the student's effort. If the student applies the maximal effort e_i , this student will get m_i points. Thus, in general, for each effort a_i , the resulting number of points g_i is equal to $g_i = a_i \cdot \frac{m_i}{e_i}$. The

student wants to minimize the overall effort $\sum_{i=1}^n a_i$ under the constraints that the overall number of points is greater than or equal to the passing value g_0 :

$\sum_{i=1}^n a_i \cdot \frac{m_i}{e_i} \geq g_0$. Thus, we arrive at the following precise formalization of the problem.

Precise formulation of the problem. Let us assume that we are given values I , e_1, \dots, e_n , and g_0 . For each tuple $m = (m_1, \dots, m_n)$ for which $\sum_{i=1}^n m_i = I$,

let $E(m)$ denote the value $\sum_{i=1}^n a_i$ corresponding to the solution to the following constraint optimization problem:

$$\sum_{i=1}^n a_i \rightarrow \min_{a_1, \dots, a_n} \quad (1)$$

under the constraints

$$0 \leq a_i \leq e_i \quad (2)$$

and

$$\sum_{i=1}^n a_i \cdot \frac{m_i}{e_i} \geq g_0. \quad (3)$$

Our goal is to select a tuple m for which the corresponding overall effort $E(m)$ is the largest possible:

$$E(m) \rightarrow \max_m. \quad (4)$$

Comment. This problem is an example of *bilevel* optimization problems, in which on the top level, we select the parameters of the objective functions so that the solution to the resulting *low-level* optimization problem will optimize an appropriate high-level objective function; see, e.g., [1].

In our case, the low level optimization is performed by a student, who is trying to minimize his/her efforts under the constraint that his overall number of points is at least g_0 . The grades m_i for each task are parameters in this student's optimization problem. The instructor – top-level optimizer – would like to select these parameters in such a way that the resulting overall student's effort is as large as possible.

2 Solution to the Problem

Description of the solution. As we show in this section, the optimal solution is to assign grades m_i proportional to the effort, i.e., to have

$$m_i = \frac{I}{E} \cdot e_i. \quad (5)$$

Proof. For the above assignment, we have $\frac{m_i}{e_i} = \frac{I}{E}$, so for all possible actual efforts a_i , the resulting grade is equal to $\sum_{i=1}^n \frac{m_i}{e_i} \cdot a_i = \frac{I}{E} \cdot \sum_{i=1}^n a_i$ and is, thus, proportional to the overall effort $A = \sum_{i=1}^n a_i$. The student wants to minimize the overall effort under the condition that this grade is at least g_0 . The corresponding constraint $\frac{I}{E} \cdot A \geq g_0$ is equivalent to $A \geq g_0 \cdot \frac{E}{I}$. Thus, the smallest possible value $E(m)$ of the overall effort A is equal to

$$E(m) = g_0 \cdot \frac{E}{I}. \quad (6)$$

Let us prove that for any other grade assignment $m' \neq m$, we have

$$E(m') < E(m) = g_0 \cdot \frac{E}{I}.$$

Indeed, the assignment m is characterized by the fact that for this assignment, the ratio $\frac{m_i}{e_i}$ is constant. Since $m' \neq m$, for this new assignment, the ratio $\frac{m'_i}{e_i}$ is not constant, it takes at least two different values for some i .

If we had $\frac{m'_i}{e_i} \leq \frac{I}{E}$ for all i , then, since $m' \neq m$, we should have $\frac{m'_j}{e_j} < \frac{I}{E}$ for some j . In this case, we have $m'_i \leq e_i \cdot \frac{I}{E}$ for all i and $m'_j < e_j \cdot \frac{I}{E}$ for some j . By adding all these inequalities, we get $\sum_{i=1}^n m'_i < E \cdot \frac{I}{E} = I$, which contradicts to the fact that for each grade assignment, we should have $\sum_{i=1}^n m'_i = I$. Thus, this case is impossible, and we have at least one index i for which $\frac{m'_i}{e_i} > \frac{I}{E}$. Let us denote one of these indices by k , then $\frac{m'_k}{e_k} > \frac{I}{E}$ and $m'_k > e_k \cdot \frac{I}{E}$. If we subtract this inequality from the equality $I = E \cdot \frac{I}{E}$, then we get $I - m'_k < (E - e_k) \cdot \frac{I}{E}$, hence $\frac{I - m'_k}{E - e_k} < \frac{I}{E}$. From this inequality and the inequality $\frac{m'_k}{e_k} < \frac{I}{E}$, we conclude that $\frac{I - m'_k}{E - e_k} < \frac{m'_k}{e_k}$. Taking the inverse of both sides, we conclude that

$$\frac{e_k}{m'_k} < \frac{E - e_k}{I - m'_k},$$

thus

$$e_k < m'_k \cdot \frac{E - e_k}{I - m'_k}. \quad (5)$$

For each $\varepsilon > 0$ and $\delta > 0$, let the student spend a little bit more effort on the k -th assignment than in the proportional assignment, i.e., $a_k = \left(\frac{g_0}{I} + \varepsilon\right) \cdot e_k$, while for all other tasks $i \neq k$, the student will spend a little less effort $a_i = \left(\frac{g_0}{I} - \delta\right) \cdot e_i$. Under the grade assignment m' , the student's grade g will be equal to

$$g = \left(\frac{g_0}{I} + \varepsilon\right) \cdot m'_k + \sum_{i \neq k} \left(\frac{g_0}{I} - \delta\right) \cdot m'_i.$$

Opening parentheses, combining terms proportional to $\frac{g_0}{I}$, and taking into account that $m_k + \sum_{i \neq k} m'_i = I$ and thus $\sum_{i \neq k} m'_i = I - m'_k$, we conclude that

$$g = \frac{g_0}{I} \cdot I + \varepsilon \cdot m'_k - \delta \cdot (I - m'_k) = g_0 + \varepsilon \cdot m'_k - \delta \cdot (I - m'_k).$$

We can get $g = g_0$ if we select δ in such a way that $\varepsilon \cdot m'_k - \delta \cdot (I - m'_k) = 0$. Then,

$$\delta = \varepsilon \cdot \frac{m'_k}{I - m'_k}. \quad (8)$$

For this selection of δ , the student's overall effort is equal to

$$A = \sum_{i=1}^n a_i = a_k + \sum_{i \neq k} a_i = \left(\frac{g_0}{I} + \varepsilon\right) \cdot e_k + \sum_{i \neq k} \left(\frac{g_0}{I} - \delta\right) \cdot e_i.$$

Opening the parentheses, combining terms proportional to $\frac{g_0}{I}$, and taking into account that $e_k + \sum_{i \neq k} e_i = E$ and thus $\sum_{i \neq k} e_i = E - e_k$, we conclude that

$$A = g_0 \cdot \frac{E}{I} + \varepsilon \cdot e_k - \delta \cdot (E - e_k).$$

According to the formula (4), the first term in the right-hand side is exactly $E(m)$ for the above grade assignment m . Substituting the expression (8) for δ into this formula, we conclude that

$$A = E(m) + \varepsilon \cdot \left(e_k - \frac{m'_k}{I - m'_k} \cdot (E - e_k) \right).$$

Due to (7), we have $A < E(m)$. By definition, $E(m')$ is the smallest possible effort the student needs to spend to get g_0 , thus $E(m') \leq A$ and hence,

$$E(m') < E(m).$$

The optimality of the grade assignment m is thus proven.

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References

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