Decision Theory Explains “Telescoping Effect” – That Our Time Perception Is Biased

Laxman Bokati and Vladik Kreinovich
Computational Science Program
University of Texas at El Paso
El Paso, TX 79968, USA
lbokati@miners.utep.edu, vladik@utep.edu

Abstract

People usually underestimate time passed since distant events, and overestimate time passed since recent events. There are several explanations for this “telescoping effect”, but most current explanations utilize specific features of human memory and/or human perception. We show that the telescoping effect can be explained on a much basic level of decision theory, without the need to invoke any specific ways we perceive and process time.

1 Formulation of the Problem

Telescoping effect. It is known that when people estimate how long ago past events happened, their estimates are usually biased (see, e.g., [1, 4, 11]):

- for recent events, people usually *overestimate* how much time has passed since this event;
- on the other hand, for events in the more distant past, people usually *underestimate* how much time has passed since the event.

This phenomenon is called *telescoping effect* since the bias in perceiving long-ago past events is similar to what happens when look at the celestial objects via a telescope: all the objects appear closer than when you look at them with a naked eye.

How can this effect be explained. There are many explanations for the telescope effect [1, 4, 11], but most current explanations utilize specific features of human memory and/or human perception.

What we do in this paper. In this paper, we show that the telescoping effect can be explained on a much basic level of decision theory, without the need to invoke any specific ways we perceive and process time.

2 Decision Theory: A Brief Reminder

What is decision theory. Decision theory (see, e.g., [2, 6, 8, 9, 10]) describes the behavior of a rational person – i.e., a person who, e.g., when deciding that an alternative A is preferable to alternative B and B is preferable to C, selects A if presented with two choices: A and C.

Real-life people are not always that rational. This well-known deviations from rationality are usually caused by our limited ability to process information in a short time needed to make a decision – this is known as *bounded rationality*; see, e.g., [5, 7]. However, overall, decision theory provides a reasonably accurate description of human behavior.

Utility: the main notion of decision theory. In decision theory, human preferences are described in terms of a special notion of *utility*. Utility can be described as follows. We select:

- a very bad alternative A_- (much worse than anything that we will actually encounter) and
- a very good alternative A_+ (much better than what we will actually encounter).

Then, for every value p from the interval $[0, 1]$, we can form a lottery $L(p)$ in which we get A_+ with probability p and A_- with the remaining probability $1 - p$. For any realistic alternative A:

- for $p = 1$, the lottery $L(p)$ is equivalent to A_+ and is, thus, better than A: $A < A_+ = L(1)$; while
- for $p = 0$, the lottery $L(p)$ is equivalent to A_- and is, thus, worse than A: $A_- = L(0) < A$.

The larger p, the better the lottery $L(p)$. Thus, there exists a threshold value u at which we switch from "$L(p)$ is worse than A" to "$L(p)$ is better than A, i.e., a value u such that:

- for $p < u$, we have $L(p) < A$, and
- for $p > u$, we have $A < L(p)$.

This value $u = u(A)$ is called the *utility* of the alternative A. In a reasonable sense, the alternative A is thus equivalent to the lottery $L(u(A))$.

The higher the utility value, the better the alternative – since each alternative A is equivalent to the lottery $L(p)$ with $p = u(A)$ and, the higher the probability p the good event A_+ in such a lottery, the better the lottery. So, among several alternatives, we should select the one with the highest utility value.

Utility is defined modulo linear transformations. The numerical value of utility depends on the selection of the alternatives A_- and A_+. It can be shown that if we select a different pair (A'_-, A'_+), then the corresponding utility $u'(A)$ is related to the original utility by a linear transformation $u'(A) = a \cdot u(A) + b$ for some $a > 0$ and b; see, e.g., [6, 9].

Decision making under interval uncertainty. In real life, we rarely know the exact consequences of each action. As a result, for each alternative A, instead of the exact value of its utility, we often only know the bounds $\underline{u}(A)$ and $\overline{u}(A)$ on this unknown value. In other words, all we know is the interval $[\underline{u}(A), \overline{u}(A)]$ that contains the actual (unknown) value $u(A)$. How can we make a decision under this interval uncertainty?

In particular, for such an interval case, we need to be able to compare the interval-valued alternative with lotteries $L(p)$ for different values p. As a result of such comparison, we will come up with a utility of this interval. So, to make recommendations on decision under interval uncertainty, we need to be able to assign, to each interval $[\underline{u}, \overline{u}]$, a single utility value $u(\underline{u}, \overline{u})$ from this interval that describes this interval's utility.

Since utility is defined modulo a linear transformation $u \to u' = a \cdot u + b$, it is reasonable to require that the corresponding function $u(\underline{u}, \overline{u})$ should also be invariant under such transformations, i.e., that:

- if $u = u(\underline{u}, \overline{u})$,
- then $u' = u(\underline{u}', \overline{u}')$, where we denoted $u' = a \cdot u + b$, $\underline{u}' = a \cdot \underline{u} + b$, and $\overline{u}' = a \cdot \overline{u} + b$.

It turns out that this invariance requirement implies that

$$u(\underline{u}, \overline{u}) = \alpha_H \cdot \overline{u} + (1 - \alpha_H) \cdot \underline{u})$$

for some $\alpha_H \in [0, 1]$ [6, 9]. This formula was first proposed by a future Nobelist Leo Hurwicz and is, thus, known as the Hurwicz optimism-pessimism criterion [3, 8].

Theoretically, we can have values $\alpha_H = 0$ and $\alpha_H = 1$. However, in practice, such values do not happen:

- $\alpha_H = 1$ would correspond to a person who only takes into account the best possible outcome, completely ignoring the risk of possible worse situations;
- similarly, the value $\alpha_H = 0$ would correspond to a person who only takes into account the worst possible outcome, completely ignoring the possibility of better outcomes.

In real life, we thus always have $0 < \alpha_H < 1$.

How to take time into account. An event – e.g., a good dinner – a year in the past does not feel as pleasant to a person now as it may have felt a year ago. Similarly, a not-so-pleasant event in the past – e.g., a painful inoculation – does not feel as bad now if it felt a year ago, when it actually happened. Thus, the utility of an event changes with time: positive utility decreases, negative utility increases (i.e., gets closer to 0). If u is the utility of a current event, how can we describe the utility $f(u)$ of remembering the same event that happened 1 year ago?

We can normalize the utility values by assuming that the status quo situation has utility 0. Then the only remaining transformation is re-scaling $u' = a \cdot u$. Similarly to the case of interval uncertainty, it is reasonable to require that the function $f(u)$ is invariant with respect to such a transformation, i.e., that:

- if we have $v = f(u)$,
- then for each a, we should have $v' = f(u')$, where we denoted $v' = a \cdot v$ and $u' = a \cdot u$.

Substituting the expressions for v' and u' into the formula $v' = f(u')$, we conclude that $a \cdot v = f(a \cdot u)$, i.e., $a \cdot f(u) = f(a \cdot u)$. Substituting $u = 1$ into this formula, we conclude that $f(a) = q \cdot a$, where we denoted $q \stackrel{\text{def}}{=} f(1)$. Since $f(u) < u$ for $u > 0$, this would imply that $q < 1$.

So, an event with then-utility u that occurred 1 year ago has the utility $q \cdot u$ now. Similarly, an event with utility u that happened 2 years ago is equivalent to $q \cdot u$ a year ago, and thus, is equivalent to $q \cdot (q \cdot u) = q^2 \cdot u$ now. We can similarly conclude that an event with utility u that occured t moments in the past is equivalent to utility $q^t \cdot u$ now.

3 How Decision Theory Can Explain the Telescoping Effect

People's perceptions are imprecise. In the ideal situation, an event of utility u_0 that occurred t moments in the past should be equivalent to exactly the utility $u = q^t \cdot u_0$ now. In practice, however, people's perceptions are imprecise.

Let us describe this imprecision: first approximation. Let us denote by ε the accuracy of people's perception. Then, for an event with actual utility u, the perceived utility can differ by ε, i.e., it can take any value from the corresponding interval $[u - \varepsilon, u + \varepsilon]$. In particular, our perceived utility u of the past event can take any value from the interval $[q^t \cdot u_0 - \varepsilon, q^t \cdot u_0 + \varepsilon]$.

How we perceive events form the distant past. The above interval can be somewhat narrowed down if we take into account that for a positive event, with utility $u_0 > 0$, the perception cannot be negative, while the value $q^t \cdot u_0 - \varepsilon$ is negative for large t. Thus, when $q^t \cdot u_0 - \varepsilon < 0$, i.e., when $t > T_0 \stackrel{\text{def}}{=} \dfrac{\ln(u_0/\varepsilon)}{|\ln(q)|}$, the lower bound of the interval is 0, and thus, the interval has the form

$$[\underline{u}, \overline{u}] = [0, q^t \cdot u_0 + \varepsilon].$$

Based on Hurwicz's optimism-pessimism criterion, this interval is equivalent to the value $\alpha_H \cdot (q^t \cdot u_0 + \varepsilon)$. How does this translate into a perceived time? For any time t_p, the utility of the event t_p moments in the past is equal to $q^{t_p} \cdot u_0$. Thus, the perceived time t_p can be found from the condition that the utility $\alpha_H \cdot (q^t + \varepsilon)$ is equal to $q^{t_p} \cdot u_0$. This equality $\alpha_H \cdot (q^t \cdot u_0 + \varepsilon) = q^{t_p} \cdot u_0$ implies that

$$t_p = \frac{\ln((\alpha_H \cdot (q^t \cdot u_0 + \varepsilon))/u_0)}{\ln(q)}.$$

In particular, when t tends to infinity, we have $q^t \to 0$ and thus, the perceived time tends to a finite constant

$$\frac{\ln((\alpha_H \cdot \varepsilon)/u_0)}{\ln(q)}.$$

Thus, for large t we indeed have $t_p \ll t$, which is exactly what we observe in the telescoping effect for events from the distant past.

How we perceive very recent events. For recent events, the interval

$$[q^t \cdot u_0 - \varepsilon, q^t \cdot u_0 + \varepsilon]$$

can also be somewhat narrowed down if we take into account that the perceived utility of a past event cannot exceed its utility now, i.e., the value u_0. Thus, when $q^t \cdot u_0 + \varepsilon > u_0$, i.e., when $q^t > 1 - \varepsilon/u_0$ and thus, $t < t_0 \stackrel{\text{def}}{=} \dfrac{\ln(1 - u_0/\varepsilon)}{\ln(q)}$, the upper bound of the interval is 1, and thus, the interval has the form

$$[\underline{u}, \overline{u}] = [q^t \cdot u_0 + \varepsilon, u_0].$$

Based on Hurwicz's optimism-pessimism criterion, this interval is equivalent to the value $\alpha_H \cdot u_0 + (1 - \alpha) \cdot (q^t + \varepsilon)$. Similarly to the distant-past case, the perceived time t_p can be found from the condition that the above value is equal to $q^{t_p} \cdot u_0$, i.e., that

$$\alpha_H \cdot u_0 + (1 - \alpha) \cdot (q^t + \varepsilon) = q^{t_p} \cdot u_0.$$

This implies that

$$t_p = \frac{\ln(\alpha_H \cdot u_0 + (1 - \alpha) \cdot (q^t + \varepsilon))}{\ln(q)}.$$

In particular, when t tends to 0, we have $q^t \to 1$ and thus, the perceived time t_p tends to a finite positive constant

$$\frac{\ln(\alpha_H \cdot u_0 + (1 - \alpha) \cdot (1 + \varepsilon))}{\ln(q)}.$$

Thus, for small t, we indeed have $t_p \gg t$, which is exactly what we observe in the telescoping effect for recent events.

Acknowledgments

This work was supported in part by the US National Science Foundation via grant HRD-1242122 (Cyber-ShARE Center of Excellence).

References

[1] S. E. Crawley and L. Pring, "When did Mrs. Thatcher resign? The effects of ageing on the dating of public events", *Memory*, 2000, Vol. 8, No. 2, pp. 111–121.

[2] P. C. Fishburn, *Utility Theory for Decision Making*, John Wiley & Sons Inc., New York, 1969.

[3] L. Hurwicz, *Optimality Criteria for Decision Making Under Ignorance*, Cowles Commission Discussion Paper, Statistics, No. 370, 1951.

[4] S. M. J. Janssen, A. G. Chessa, and J. M. J. Murre, "Memory for time: how people date events", *Memory and Cognition*, 2006, Vol. 34, No. 1, pp. 138–147.

[5] D. Kahneman, *Thinking, Fast and Slow*, Farrar, Straus, and Giroux, New York, 2011.

[6] V. Kreinovich, "Decision making under interval uncertainty (and beyond)", In: P. Guo and W. Pedrycz (eds.), *Human-Centric Decision-Making Models for Social Sciences*, Springer Verlag, 2014, pp. 163–193.

[7] J. Lorkowski and V. Kreinovich, *Bounded Rationality in Decision Making Under Uncertainty: Towards Optimal Granularity*, Springer Verlag, Cham, Switzerland, 2018.

[8] R. D. Luce and R. Raiffa, *Games and Decisions: Introduction and Critical Survey*, Dover, New York, 1989.

[9] H. T. Nguyen, O. Kosheleva, and V. Kreinovich, "Decision making beyond Arrow's 'impossibility theorem', with the analysis of effects of collusion and mutual attraction", *International Journal of Intelligent Systems*, 2009, Vol. 24, No. 1, pp. 27–47.

[10] H. Raiffa, *Decision Analysis*, McGraw-Hill, Columbus, Ohio, 1997.

[11] Wikipedia, *Telescoping effect*, https://en.wikipedia.org/wiki/Telescoping_effect, downloaded on December 29, 2018