In the Absence of Information, $1/n$ Investment Makes Perfect Sense

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Abstract

When people have several possible investment instruments, people often invest equally into these instruments: in the case of n instruments, they invest $1/n$ of their money into each of these instruments. Of course, if additional information about each instrument is available, this $1/n$ investment strategy is not optimal. We show, however, that in the absence of reliable information, $1/n$ investment is indeed the best strategy.

1 $1/n$ Investment: Formulation of the Problem

General investment problem. People saving for retirement usually have several options to invest: they can invest in stocks, they can invest in bonds, they can invest in funds that combine stocks and bonds, etc. An important decision is how to allocate money between different financial instruments, i.e., how much money should we invest in each of the instruments.

Markowitz's solution. A solution to this problem was proposed in 1952 by the future Nobelist H. M. Markowitz [4]. He actually solved two different versions of the investment problem:

- the first version is when we want to achieve a certain expected growth rate, and within this expected rate, we select an investment portfolio that minimizes the risk (as measured by the standard deviation of the growth rate);
- the second version is when we fix the risk level, and within this risk level, we select an investment portfolio that maximizes the expected growth rate.

What is $1/n$ investment. In the absence of reliable information, people tend to divide their investment amount equally between different investment options:

if there are n investment options, they invest exactly $1/n$ of the original amount in each of these options. This practice is known as the $1/n$ *investment*; see, e.g., [1, 9].

Numerous experiments confirm that this is how, in the absence of detailed information, people invest their money [1, 2, 5, 8, 9]. We are not talking only about common folks who do not understand the corresponding economic details: this is, e.g., how Markowitz himself invested his retirement money [9, 10].

Why? A natural question is: why is this strategy so ubiquitous? This question is formulated, e.g., in [9] – this book tends to use the ubiquity of this strategy as one of the arguments that people do not always behave rationally.

In this paper, we provide a natural and simple explanation for this strategy – which shows that at least in this case, people do behave rationally.

2 Our Explanation

Main idea. The absence of information about the available investment options means that we have no reason to assume that one of them is better than the other. In other words, based on the available information (or, to be more precise, based on the absence of any information), the situation is completely invariant with respect to any permutation of the available n options.

It is therefore reasonable to conclude that the resulting allocation of the available money amount between different investment options should also be invariant with respect to the same transformations.

This idea explains the $1/n$ investment strategy. Allocating funds means selecting, for each investment option i, the proportion $p_i \geq 0$ that will be invested in this particular option. The only a priori restriction is that all the money should be invested, i.e., that these proportions should all up to 1: $p_1 + \ldots + p_n = 1$.

Our requirement is that the allocation should be invariant with respect to any permutation $\pi : \{1, \ldots, n\} \to \{1, \ldots, n\}$. This means, in particular, that for every i and j, the allocation $p_1, \ldots, p_n$ should be invariant with respect to the permutation that swaps i and j and leaves all other elements unchanged. This invariance means that we should have $p_i = p_j$. Since this should be true for all i and j, all n allocations $p_1, \ldots, p_n$ should be equal to each other. Since their sum should be equal to 1, this means that each of them should be equal to $1/n$ – thus, in the absence of information, the $1/n$ investment is indeed the most reasonable strategy.

3 Discussion

What about probabilistic investment strategies? In the previous section, we considered only *deterministic* investment strategies, in which we select the allocations p_i. However, in principle, we can have *probabilistic* investment strategies, in which, instead of selecting an allocation vector $p = (p_1, \ldots, p_n)$ from the very beginning, we could:

- select a probability distribution on the set of all allocation vectors p, and then
- when it comes to each real investment opportunity, select one of the vectors p with the corresponding probability.

Why probabilistic strategies make sense. At first glance, this may seem like a purely mathematical exercise, but the experience of game theory has shown that in many situation, such a probabilistic choice works much better than any deterministic one; see, e.g., [3].

This fact is easy to explain. As an example, let us consider a simpler robbers-and-cop situation, when robbers want to rob one of the two banks, and a local police department only has enough folks to protect a single bank. In this case, if the police department selects a deterministic strategy – i.e., allocated all its resources into one of the banks all the time – this will be a disaster: robbers will know which bank is protected and thus rob the other bank. The only strategy avoiding such a disaster is a probabilistic one, in which each time, the police officers are allocated to each bank with probability 1/2. Then, the robbers have a 50% chance of being caught no matter which bank they select, and this will most probably deter them from attempting an attack.

Probabilistic strategies reduce to deterministic ones. Suppose that we have a probabilistic strategy for investment. This strategy is optimal for any possible investment value.

Suppose now that we have an amount x that we want to invest. One possibility is to use the optimal probabilistic strategy to invest the whole amount. However, alternatively, we can divide the original amount x into two smaller amounts $x/2$ each. For each of these two amounts, we can use the same optimal probabilistic investment strategy. In this case, the overall investment is equal to the sum of two independent identically distributed random variables.

Instead of dividing the investment amount into two equal parts, we can similarly divide it into N equal parts, for each integer n. In this case, the resulting investment is the sum of N independent identically distributed random variables.

For large N, the Law of Large Numbers applies (see, e.g., [7]), so the resulting distribution will simply concentrate on the expected allocation for each of the instruments – i.e., in this case, due to symmetry, on the allocation in which we assign the same proportion $1/n$ to each of these instruments. Thus, if a probabilistic strategy is optimal, the corresponding deterministic $1/n$ investment strategy is also optimal.

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