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S. Aguilar and V. Kreinovich
University of Texas at El Paso, El Paso, Texas, USA

WHY QUANTILES ARE A GOOD DESCRIPTION OF VOLATILITY IN ECONOMICS

What is volatility. One of the main problems in economics is how to best invest money. Money can be invested in different financial instruments: stocks, bonds, derivatives, etc.

- For some of these instruments – e.g., for some bonds – we know exactly how much interest we will get in the next year.
- For other instruments, e.g., for stocks, we may gain a lot – or we may even lose, if the price of this stock goes down.

In economics, this unpredictability of gain is known as *volatility*.

Comment. Usually, the riskier instruments – i.e., instruments with higher volatility – provide better expected return: otherwise, why would someone invest in a risker instrument if it is possible to invest if a less risky instrument with the same (or even better) expected return on investment.

How can we describe volatility in numerical terms. To make investment decisions, we need to be able to compare the risks of different instruments. For this, we need to be able to describe volatility in numerical terms.

Originally, volatility was described in terms of the standard deviation of the return. However, it turned out that standard deviation does not always adequately describe our intuitive idea of volatility; sometimes:

- instruments with higher standard deviations are perceived as less risky, and,
- vice versa, instruments with lower standard deviations are perceived as more risky.

It turns out that a better description of intuitive volatility is provided by *quantiles* of the distribution – i.e., the values x for which the probability that the return r is smaller than x is equal to some selected value p. For example:

- for p = 1 / 2, we get a median;
- for p = 1 / 4 and p = 3 / 4, we get quartiles, etc.

Comment. Quantiles are usually defined in terms of the *cumulative distribution function* (cdf) F(x) = Prob(X < x), as the corresponding value $F^{--1}(p)$ of the inverse function.

Natural question: why quantiles? A natural question is: why namely quantiles – and not any other characteristics of a probability distribution – provide a description of volatility which is the closest to the intuitive understanding of volatility.

In this paper, we provide an explanation.

Main idea. In our explanation, we use the fact that the effect of money on a person's happiness is unusually nonlinear.

This fact is easy to explain:

- if you have no money at all, and you gain a dollar, you are happy, but
- if you already have $1,000 and you gain one more dollar, you will barely notice the difference.

Psychological experiments show that this nonlinear dependence of happiness on money is somewhat different for different people.

How is this related to the intuitive idea of volatility? When people think of volatility, they think in terms of changes in their happiness level – which is related to money by some non-linear transformation.

So, what we want is a characteristic that would not change if we consider a different person, with a different non-linear function relating money and happiness level. In other words, as a volatility characteristic, we want to have a characteristic c(X) of a random variable X (that describes possible gains) to have the following property:

- if the volatility characteristic measured in the money scale is c(X),
- then for any monotonic re-scaling function f(x), the volatility c(Y) of a re-scaled variable Y = f(X) should be equal to the same value c(X) described in the new scale, i.e., we should have

$$c(f(X))=f(c(X)).$$

This natural invariance requirement leads to quantiles. Let us show that this natural invariance requirement explains the appearance of quantiles. Indeed, it is known that if we have the cdf F(x), then the variable Y=F(X) is uniformly distributed on the interval [0,1].

Let p denote the value of the desired characteristic c(U) for the case when the random variable is uniformly distributed on the interval [0,1]. Since F(X) = U, we have c(F(X)) = p. Thus, the above invariance requirement takes the form c(F(X)) = F(c(X)), i.e., the form p = F(c(X)). By applying the inverse function $F^{-1}(z)$ to both sides of this equality, we conclude that $c(X) = F^{-1}(p)$.

This is exactly the quantile value that we tried to explain.

Conclusion. We have indeed explained why quantiles are the best description of intuitive notion of volatility.