

Why Strings, Why Quark Confinement: A Simple Qualitative Explanation

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Abstract

In this pedagogical article, we recall the infinities problem of modern physics, and we show that the natural way to overcome this problem naturally leads to strings and to quark confinement.

1 Physics' Problem with Infinities: A Brief Reminder

Description of the problem. One of the problems with quantum field theories is that for many physical properties, we get physically meaningless infinite values; see, e.g., [1, 2]. This problem did not start with quantum physics, it is present in non-quantum field theories as well. Following [1], let us briefly describe this problem on the example of estimating the overall energy of the electric field generated by a charged elementary particle – e.g., by an electron.

The fact that the electron is an *elementary* particle means that it does not consist of any sub-particles, i.e., it does not have any parts that act independently. Taking into account special relativity theory, with its requirement that nothing can travel faster than the speed of light, this implies that an electron must be a point-wise particle: otherwise, if it was spread over at least two different locations, there would be no way for one location to immediately influence another one, so these two locations would constitute, in effect, two different sub-particles.

The electric field $\vec{E}(\vec{x})$ of a point-wise charged particle follows Coulomb's law $|\vec{E}(\vec{x})| = \text{const} \cdot \frac{1}{r^2}$, where r is the distance from the particle. It is known that the energy density $\rho(\vec{x})$ of the electric field at each spatial location $\vec{x}$ is proportional to $(\vec{E}(\vec{x}))^2$, and thus, $\rho(\vec{x}) = \text{const} \cdot \frac{1}{r^4}$. To find the overall energy

E of this field, we can integrate this density over the whole space:

$$E = \int \rho(\vec{x}) d\vec{x} = \text{const} \cdot \int \frac{1}{r^4} d\vec{x}.$$

In the radial coordinate system centered on the electron, we get

$$E = \text{const} \cdot \int_0^\infty \frac{1}{r^4} \cdot 4\pi \cdot r^2 dr = \text{const} \cdot \int_0^\infty \frac{1}{r^2} dr. \quad (1)$$

The integral of the expression in the right-hand side is equal to $-\text{const} \cdot \frac{1}{r}$; so, the energy is equal to the difference between the values of this expression at 0 and at infinity:

$$E = -\text{const} \cdot \left(\frac{1}{\infty} - \frac{1}{0} \right).$$

Here, $\frac{1}{\infty} = 0$ and the ratio $\frac{1}{0}$ is infinite, so we conclude that the overall energy is infinite – and moreover, that there is infinite energy in any small vicinity of the electron.

If there was such an energy, then, due to another fact from special relativity – that $E = m \cdot c^2$ – we would have an infinite mass, and this infinite mass would cause infinite gravitational field – something we clearly do not observe.

Quantum effects do not help much. The above arguments assume that the usual formulas of physics are valid for all possible distances r , no matter how small. In quantum physics, there is a length $r_0 \approx 10^{-35}$ m – called Planck length – below which clearly quantum space-time effects cannot be ignored and thus, all the physics will be different. So, a natural idea is to only consider distances up to $r = r_0$, i.e., replace the integral (1) with the value

$$E = \text{const} \cdot \int_{r_0}^\infty \frac{1}{r^2} dr. \quad (2)$$

This integral is equal to $E = \text{const} \cdot \frac{1}{r_0}$. This value is no longer infinite, but because the Planck's length is so small, this value is unrealistically high – clearly, the electron does not have that much mass.

So, what can we do?

2 Qualitative Idea and Its Consequences

The idea. The infinite (or, alternatively, too large) values come from the need to consider electron as a point-wise particle, i.e., in mathematical terms, as a 0-dimensional particle. So, a natural idea is to relax his assumption and consider higher-dimensional models of elementary particles.

The closest to 0-dimensional are 1-dimensional particles, then 2-dimensional, finally fully 3-dimensional ones. Let us consider what will be the effect of 1- and 2-dimensional particles.

Where $\frac{1}{r^2}$ dependence comes from. For a point-wise particle, the effect of the field spreads out. By the time this effect reaches distance r , it has been divided by the area of the sphere of radius r , i.e., by $4\pi \cdot r^2$. This explains why the electric field of a point-wise particle is inverse proportional to r^2 .

Similar arguments will help us find the formulas for the electrical field of 1-D and 2-D particles.

1-D particles. For a 1-D particle of length L – which locally looks like a straight line – the area of distance r from the particle is a cylinder, with the surface $L \cdot 2\pi \cdot r$. Thus, the electric field is inverse proportional to r , i.e., $|\vec{E}(\vec{x})| = \text{const} \cdot \frac{1}{r}$, and the energy density $\rho(\vec{x}) = \text{const} \cdot \left(\vec{E}(\vec{x})\right)^2$ is proportional to $\frac{1}{r^2}$.

In the cylindrical coordinates, the overall energy $E = \int \rho(\vec{x}) d\vec{x}$ is equal to

$$E = \text{const} \cdot \int \frac{1}{r^2} d\vec{x} = \text{const} \cdot L \cdot \int_0^\infty \frac{1}{r^2} \cdot 2\pi \cdot r dr = \text{const} \cdot \int_0^\infty \frac{1}{r} dr.$$

This integral is proportional to $\ln(r)$, so the overall energy is proportional to the difference $\ln(\infty) - \ln(0)$, i.e. still to infinity.

For large r , we cannot use this formula – for a localized particle, when the distance is sufficiently larger than its size, the field is the same as for the point-wise particle, so we can replace infinity with the particle spatial size s . So, strictly speaking, we get $\ln(s) - \ln(0)$, still the infinity.

However, if we take into account the quantum effects and replace the lower integration bound by r_0 , we get $\ln(s) - \ln(r_0)$. Good news is that even for very small value r_0 , its logarithms is quite a reasonable number – so this is a physical meaningful idea; see, e.g., [3].

The main restriction is that the particle should be 1-D in the vicinity of each its location, i.e., it cannot have endpoints – otherwise we would have the same infinity as in the pointwise case. So, if we are interested in particles located in a small spatial region, a particle must form a closed loop – and this is exactly what *strings* are. Thus, this idea explains, on the qualitative level, why strings appear in quantum physics, and why they are so successful in eliminating infinities.

2-D particles. For a 2-D particle of area A – which locally looks like a small piece of a plane – the area of distance r from the particle is two parallel planes at this distance, at two sides of the particle – of total area $2A$. Thus, locally, the field is simply constant. Thus, the overall density is constant, and the energy is finite.

The constant field leads to the constant force field $F = \text{const}$, which, in turn, has the following side effect: that to get separated by a distance d , we need to spend the energy $F \cdot d$. Thus, no matter how much initial energy E_0 particles have, they will never fully separate: their separation will only reach the distance $d = E_0/F$ and then stop. This is exactly what is observed with quarks under the name of *quark confinement*: they can get closer to each other

or further away from each other, but they can never separate from each other – that is why we can see them, e.g., inside protons (as so-called *partons*) – but we cannot observe a free quark.

Acknowledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and HRD-1834620 and HRD-2034030 (CAHSI Includes).

References

- [1] R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Addison Wesley, Boston, Massachusetts, 2005.
- [2] K. S. Thorne and R. D. Blandford, *Modern Classical Physics: Optics, Fluids, Plasmas, Elasticity, Relativity, and Statistical Physics*, Princeton University Press, Princeton, New Jersey, 2017.
- [3] F. Zapata, O. Kosheleva, and V. Kreinovich, “Logarithms are not infinity: a rational physics-related explanation of the mysterious statement by Lev Landau”, *Proceedings of the World Congress of the International Fuzzy Systems Association and the Annual Conference of the North American Fuzzy Information Processing Society IFSA/NAFIPS’2019*, Lafayette, Louisiana, June 18–22, 2019, pp. 746–751.