

A Possible (Qualitative) Explanation of the Hierarchy Problem in Theoretical Physics

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Abstract

One of the important open problem in theoretical physics is the hierarchy problem: how to explain that some physical constant are many orders of magnitude larger than others. In this paper, we provide a possible qualitative explanation for this phenomenon.

1 Formulation of the Problem

The problem. One of the important open problems in physics – known as the *hierarchy problem* – is to explain why there is a such a huge difference between physical constants.

For example, gravitational forces are 10^{40} times weaker than electromagnetic forces – and this difference is mirrored in the difference between the corresponding constants; see, e.g., [1, 3].

What we do in this paper. In this paper, we provide a possible qualitative explanation for this phenomenon.

2 Our Explanation

A mathematical result on which our explanation is based: formulation. Our explanation uses the following result from [4]: in a nutshell, that when we reconstruct a state based on the measurement results, we will most probably get not the original state, but one of the extreme points of the set of possible states.

Why this result is valid: an explanation. A state can be characterized by the values of several physical quantities $s_1, \dots, s_n$. In this sense, each state can be represented as a tuple $s = (s_1, \dots, s_n)$.

To determine the state, we measure the values of these quantities. Measurements are never absolutely accurate, thus the results $r_1, \dots, r_n$ of measuring these quantities are, in general, different from the actual values s_i of these quantities.

Usually, the measurement errors are independent and normally distributed, with 0 mean and some standard deviation σ . In this case, for large n , we have

$$\sum_{i=1}^n \frac{(s_i - r_i)^2}{\sigma^2} \approx n; \quad (1)$$

see, e.g., [2].

The approximate equality (1) can be described in an equivalent form

$$d^2(s, r) \approx n \cdot \sigma^2,$$

where

$$d(s, r) \stackrel{\text{def}}{=} \sqrt{\sum_{i=1}^n (r_i - s_i)^2}$$

is the usual (Euclidean) distance between the n -dimensional points

$$s = (s_1, \dots, s_n) \text{ and } r = (r_1, \dots, r_n).$$

Thus, here $d(s, r) \approx \varepsilon \stackrel{\text{def}}{=} \sqrt{n} \cdot \sigma$. In other words, the observed state r is located at distance ε from the actual state s .

The observed state r is, in general, outside the set S of possible states. So, to reconstruct the state s , we need to find an appropriate state a within the set S . In general, there are several such states. A natural idea is to use the Maximum Likelihood method – i.e., to select the most probable state. For the normal distribution, the corresponding probability density is described by the following expression:

$$\prod_{i=1}^n \left(\frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot \exp \left(-\frac{(r_i - a_i)^2}{2\sigma^2} \right) \right).$$

This expression can be equivalently reformulated as

$$\frac{1}{(\sqrt{2\pi} \cdot \sigma)^n} \cdot \exp \left(-\frac{1}{2\sigma^2} \cdot \sum_{i=1}^n (r_i - a_i)^2 \right).$$

Thus, maximizing this expression is equivalent to minimizing the sum

$$\sum_{i=1}^n (r_i - a_i)^2,$$

i.e., equivalently, the square root of this expression:

$$d(r, a) \stackrel{\text{def}}{=} \sqrt{\sum_{i=1}^n (r_i - a_i)^2}.$$

Thus, when we reconstruct the signal based on the measurement results r , we select the state $a \in S$ which is the closest to the tuple r that describes the measurement results.

Let us show how this geometric fact leads to the above conclusion. For simplicity, let us consider the case when $n = 2$ and the set S of possible states is a convex polygon. The point $r = (r_1, r_2)$ formed by measurement results is then located at distance ε from the set S .

Let us look at all the points located at the distance ε from the polygon:

- for some of these points, the closest point on the polygon is located on one of its edges;
- for others, the closest point is one of the polygon's vertices.

One can check that for each edge, points for which the closest is on this edge are located on a segment parallel to this edge and bounded by the two lines orthogonal to this edge. The length of this segment is equal to the length of the edge. Thus, the overall length of these segments is equal to the sum of the lengths of all the edges, i.e., to the perimeter of the polygon. So, this overall length does not grow when ε increases.

On the other hand, the set of all the points which are ε -distant from the set S grows with ε . Thus, as ε increases, the proportion of the points r for which the closest point $a \in S$ is on one of the edges decreases – and hence, for a bigger and bigger proportion of points r , the closest point a is one of the vertices – i.e., one of the extreme points of the original set S of possible states.

Comment. Of course, these are somewhat informal arguments. For an exact formulation and proof, see [4].

How this mathematical fact helps in our explanation. In our case, we are considering the values of different physical constants. We can consider two possible situation:

- In many cases, we know the sign of the corresponding constant: for example, we know that gravity only leads to attraction between different objects.
- In other cases, we may now know the sign: e.g., electric forces can lead both to attraction and to repulsion.

In the first case, the set of possible values of the corresponding constant is the interval $(0, \infty)$. In the second case, the set of possible values of the corresponding constant is the whole real line $(-\infty, +\infty)$.

According to the above mathematical result, when we reconstruct the value of the corresponding constant from observations, most probably, we will get one of the extreme points of the corresponding set of values. For an interval, extreme points are its endpoints. Thus:

- in the first case, we will get either 0 or infinity;

- in the second case, we will get either $-\infty$ or $+\infty$.

Of course, we cannot have 0 or infinity – this does not make physical sense, so this means that we will have either a very small value or a very large value – and this is exactly what we observe, this is exactly what the hierarchy problem is about.

Thus, we have found a possible qualitative explanation for this empirical phenomenon.

Comment. Not only we found a possible explanation for the hierarchy problem in general, our arguments also explain why the constant corresponding to the electromagnetic interaction has to be large – since this interaction allows forces of both signs, attraction and repulsion.

In contrast, gravity has only interactions of one type, so it is not surprising that the corresponding constant is close to 0 in comparison to the electromagnetic one.

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