Scale-Invariance Ideas Explain the Empirical Soil-Water Characteristic Curve

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***Abstract*—The prediction of the road's properties under the influence of water infiltration is important for pavement design and management. Traditionally, this prediction heavily relied on expert estimates. In the last decades, complex empirical formulas have been proposed to capture the expert's intuition in estimating the effect of water infiltration on the stiffness of the pavement's payers. Of special importance is the effect of water intrusion on the pavement's foundation – known as subgrade soil. In this paper, we show that natural scale-invariance ideas lead to a theoretical explanation for an empirical formula describing the dependence between soil suction and water content; formulas describing this dependence are known as soil-water characteristic curves.**

***Keywords*—Expert knowledge, transportation engineering, scale invariance, soil-water characteristic curve**

I. Formulation of the Problem

Need to take into account water content in road design and management. It is important to make sure that the roads retain sufficiently stiff under all possible weather conditions. Out of different weather conditions, the most important effect on the road stiffness is produced by rain: rainwater penetrates the reinforced-soil foundation of the pavement (called *subgrade soil*) that underlies more stiff layers of the road, and the presence of water decreases the road's stiffness.

Towards the empirical formulas. The mechanical effect of water can be described by the corresponding pressure h. In transportation engineering, this pressure is known as *suction*.

This pressure is easy to explain based on every person's experience of walking on an unpaved road:

- when the soil is dry, it exerts high pressure on our feet, thus preventing shoes from sinking, and keeping the surface of the road practically intact;
- on the other hand, when the soil is wet, the pressure drastically decreases; as a result, the shoes sink into the road, and leave deep tracks.

Similarly, the car's wheels sink into a wet road and leave deep tracks. The effect is not so prominent on paved roads, but still moisture affects the road quality.

To describe this effect in quantitative terms – and thus, to predict the effect of different levels of water saturation – we need to find the relation between the water content and the suction. Usually, for historical reasons, this effect is described as the dependence of water content θ on suction h – but we can also invert this dependence and consider the dependence of suction h on the water content θ. The dependence of θ on h is known as the *soil-water characteristic curve* (SWCC, for short).

Until the 1990s, this dependence was described by a power law $\theta = c \cdot h^{-m}$ for some parameters c and $m > 0$. (Since the suction decreases with the increase in water content, the exponent $-m$ should be negative.)

This power law formula was first proposed in [2] by R. H. Brooks and A. T. Corey. Many empirical studies confirmed this dependence; see, e.g., [3], [4], [6], [7], [8], [13], [17].

This law works reasonably well for intermediate values of θ. However, this formula is not perfect. For example, for $\theta \to 0$, this formula – or, to be precise, the inverse formula $h = \text{const} \cdot \theta^{-1/m}$ – implies the physically unreasonable infinite value of suction pressure. For the important case when the soil is heavily saturated with water – i.e., when θ is large – it is also not in good accordance with the empirical data.

As a result of this imperfection, in practice, until the 1990s, the results of the above power law formula were usually corrected by experts. To get a better fit with the observations and with the expert estimates, the paper [5] by D. G. Fredlung and A. Xing proposed a more complex formula

$$\theta = \text{const} \cdot \left(\ln(e + (h/a)^b)\right)^{-c}, \quad (1)$$

for some parameters a, b, and c. This formula has been experimentally confirmed for a wide range of values of the water content θ; see, e.g., [5], [12], [18]. At present (2020), this formula – with a minor modification that we will discuss later – is recommended by the US standards; see, e.g., p. 209 of Appendix DD1 "Resilient Modulus as Function of Soil Moisture – Summary of Predictive Models" of [10] and Chapter 5, p. 42 of [9] (see also Section 2.3 of [1]).

Comment. In many applications, to get an even more accurate description, practitioners multiply the right-hand side of the formula (1) by an additional factor

$$C(h) = 1 - \frac{\ln\left(1 + \dfrac{h}{h_r}\right)}{\ln\left(1 + \dfrac{h_0}{h_r}\right)} \tag{1a}$$

for some values h_r and h_0.

How can we explain the empirical formula (1)? Empirically, the formula (1) works well, but there is no theoretical explanation for this formula. Without a theoretical explanation, we cannot guarantee that this formula is indeed the best – and that no new formula would appear which would fit the observations even better. In general, the presence of a theoretical explanation increases our confidence in an empirical formula. From this viewpoint, it is desirable to come up with a theoretical explanation for the formula (1).

Such an explanation is provided in this paper.

II. Analysis of the Problem

Scale-invariance: reminder. Let us start with a general problem of finding the dependence $y = f(x)$ between two physical quantities x and y.

From the purely mathematical viewpoint, the problem seems straightforward: we need to find the relation between the two numerical values. However, from the physical viewpoint, we need to take into account that the same physical quantity can be represented by different numerical values – the specific value depends on what measuring unit we select. For example, we can measure distances in meters or kilometers; the same distance will be represented by different numbers: 2 km becomes 2000 m. With pressure characteristics (like suction), we can use Pascals or we can the US unit psi (pounds per square inch). In general, if we replace the original measuring unit with a different unit which is $\lambda > 0$ times smaller, all numerical values get multiplied by λ: $x \to x' = \lambda \cdot x$.

In many physical situations, there is no selected measuring unit, so the formulas should not depend on what measuring unit we use. Of course, we cannot simply require that the formula remains exactly the same if we change the unit for x: that would means that $f(x) = f(\lambda \cdot x)$ for all λ and x – and thus, that $f(x) = \text{const}$, i.e., that there is no dependence at all. In reality, if we change the unit for x, we need to appropriately change the unit for y. For example, the formula $y = x^2$ for the area y of a square with side x remains valid if we switch from meters to centimeters – but then we need to also change the measuring unit for area from square meters to square centimeters.

So, the desired property takes the following form: for each $\lambda > 0$, there should exist a value $\mu > 0$ such that if $y = f(x)$, then $y' = f(x')$, where $x' \stackrel{\text{def}}{=} \lambda \cdot x$ and $y' \stackrel{\text{def}}{=} \mu \cdot y$. This property is known as *scale-invariance.*

Which dependencies are scale-invariant? Substituting $y' = \mu \cdot y$ and $x' = \lambda \cdot x$ into the formula $y' = f(x')$, we get $\mu \cdot y = f(\lambda \cdot x)$. Here, we have $y = f(x)$, so $f(\lambda \cdot x) = \mu \cdot f(x)$. Taking into account that μ depends on λ, we get the following expression:

$$f(\lambda \cdot x) = \mu(\lambda) \cdot f(x). \tag{2}$$

Small changes in x should cause equally small changes in y, so the dependence $f(x)$ must be smooth (differentiable). From the formula (2), we can conclude that the function $\mu(\lambda)$ is equal to the ratio of two differentiable functions $\mu(\lambda) = \dfrac{f(\lambda \cdot x)}{f(x)}$ and is, thus, differentiable too.

Since both functions $f(x)$ and $\mu(\lambda)$ are differentiable, we can differentiate both sides of the formula (2) with respect to λ. After plugging in $\lambda = 1$, we get

$$x \cdot \frac{df}{dx} = a \cdot f,$$

where we denoted

$$a \stackrel{\text{def}}{=} \frac{d\mu}{d\lambda}_{|\lambda=1}.$$

We can separate the variables in this formula if we divide both sides by f and by x, then we get:

$$\frac{df}{f} = a \cdot \frac{dx}{x}.$$

Integrating both sides, we get $\ln(f) = a \cdot \ln(x) + C$, where C is the integration constant. Applying the exponential function to both sides of this formula, we get $f = c \cdot x^a$, where we denoted $c \stackrel{\text{def}}{=} \exp(C)$. So, every scale-invariant dependence is a power law.

Vice versa, it is easy to show that very power law has the scale invariance property.

Need to go beyond scale-invariance. As we have mentioned earlier, historically the first formulas for describing the soil-water characteristic curves were indeed the power law formulas – and the above derivation explains why these formula provide a good first approximation. However, as we also mentioned earlier, the power law is a crude approximation, we need to go beyond power laws. How can we do that?

A natural idea is to take into account that in nature, dependencies are rarely direct: usually, when we see that a change in a quantity x leads to a change in a quantity y, this means that:

- a change in x changes some intermediate quantity x_1,
- the change in x_1, in turn, leads to the change in some other intermediate quantity x_2, etc.,
- until we finally teach some quantity x_k that directly affects y.

To describe this complex dependence, we need to describe:

- how x_1 depends on x, we will denote the corresponding dependence by $x_1 = f_1(x)$,
- how x_2 depends on x_1, we will denote the corresponding dependence by $x_2 = f_2(x_1)$, etc.,
- and how y depends on x_k, we will denote the corresponding dependence by $y = f_{k+1}(x_k)$.

Then, we have

$$y = f_{k+1}(x_k) = f_{k+1}(f_k(x_{k-1})) = \ldots =$$
$$f_{k+1}(f_k(\ldots f_2(f_1(x))\ldots)).$$

In other words, the function $f(x)$ describing the (indirect) dependence between x and y is a composition of several functions $f_1(x)$, $f_2(x_1)$, $\ldots$, $f_{k+1}(x_k)$ describing direct dependencies.

At first glance, it is reasonable to assume – as we did earlier – that all the direct dependencies are scale-invariant and are, thus, described by power laws. However, one can easily check that a composition of power laws is also a power law: indeed, e.g., if $x_1 = f_1(x) = c_1 \cdot x^{a_1}$ and $x_2 = f_2(x_1) = c_2 \cdot x_1^{a_2}$, then

$$x_2 = c_2 \cdot (c_1 \cdot x^{a_1})^{a_2} = (c_1 \cdot c_1^{a_2}) \cdot x^{a_1 \cdot a_2},$$

i.e., the dependence of x_2 on x has the form $x_2 = c \cdot x^a$, where $c = c_2 \cdot c_1^{a_2}$ and $a = a_1 \cdot a_2$. Thus, the above idea does not allow us to go beyond power laws.

To go beyond power laws, we therefore need, at least for one of the intermediate dependencies, to go beyond scale-invariance.

How to go beyond scale-invariance? Scale-invariance assumes that we have a fixed starting point for measuring a quantity. This is true for most physical quantities, but for some physical quantities, we can select different starting points. For example, for measuring temperature, we can select, as a starting point, the temperature at which water freezes – and get the usual Celsius scale – or we can select the absolute zero and thus get the Kelvin scale. For different purposes, different starting points may be more appropriate.

If we change a starting point for measuring x to a different starting point which is x_0 units smaller the original one, then this value x_0 will be added to all numerical values of this quantity: $x \to x' = x + x_0$, so that $x = x' - x_0$. Similarly, if we change a starting point for measuring y to a different starting point which is y_0 units smaller than the original one, then this value y_0 will be added to all numerical values of this quantity: $y \to y' + y_0$, so that $y = y' - y_0$.

If in the new units x' and y', we have a power law dependence $y' = c \cdot (x')^a$ (motivated by scale-invariance), then in the original units x and y, we will have

$$y = y' - y_0 = c \cdot (x')^a - y_0 = c \cdot (x + x_0)^a - y_0,$$

i.e., the form

$$y = c \cdot (x + x_0)^a - y_0. \qquad (3)$$

It is thus reasonable to replace one of the intermediate power-law dependencies with this more general formula.

Which value a should we choose for this modified intermediate dependence: general idea. In our analysis, we can use an observation (made in the 1980s by B. S. Tsirelson [16]) that in many cases, when we reconstruct the signal from the noisy data, and we assume that the resulting signal belongs to a certain class, the reconstructed signal is often an *extreme* point from this class; see also [11], [15]. The paper [16] provided the following *geometric* explanation to this fact: namely, when we reconstruct a signal from a mixture of a signal and a Gaussian noise, then the *maximum likelihood* estimation (a traditional statistical technique; see, e.g., [14]) means that we look for a signal which belongs to the (a priori determined) class of signals, and which is the closest – in the sense of the usual Euclidean distance – to the observed signal-plus-noise combination.

In particular, if the signal is determined by finitely many (say, d) parameters, we must look for a signal $\vec{s} = (s_1, \ldots, s_d)$ from the a priori set $A \subseteq R^d$ that is the closest (in the usual Euclidean sense) to the observed values

$$\vec{o} = (o_1, \ldots, o_d) = (s_1 + n_1, \ldots, s_d + n_d),$$

where n_i denotes the (unknown) values of the noise.

Since the noise is Gaussian, we can usually apply the *Central Limit Theorem* [14] and conclude that the average value of $(n_i)^2$ is close to σ^2, where σ is the standard deviation of the noise. In other words, we can conclude that

$$(n_1)^2 + \ldots + (n_d)^2 \approx d \cdot \sigma^2.$$

In geometric terms, this means that the distance

$$\sqrt{\sum_{i=1}^{d} (o_i - s_i)^2} = \sqrt{\sum_{i=1}^{d} n_i^2}$$

between $\vec{s}$ and $\vec{o}$ is $\approx \sigma \cdot \sqrt{d}$. Let us denote this distance $\sigma \cdot \sqrt{d}$ by ε.

Let us first, for simplicity, consider the case when $d = 2$, and when A is a convex polygon. Then, we can divide all points p from the exterior of A that are ε-close to A into several zones depending on what part of A is the closest to p:

- one of the *sides*, or
- one of the *edges*.

Geometrically, the set of all points for which the closest point $a \in A$ belongs to the *side* e is bounded by the straight lines orthogonal (perpendicular) to e. The total length of this set is therefore equal to the length of this particular side; hence, the total length of all the points that are the closest to all the sides is equal to the *perimeter* of the polygon. This total length thus does not depend on ε at all.

On the other hand, the set of all the points at the distance ε from A grows with the increase in ε; its length grows approximately as the length of a circle, i.e., as const$\cdot\varepsilon$.

When ε increases, the (constant) perimeter is a vanishing part of the total length. Hence, for large ε:

- the fraction of the points that are the closest to one of the sides tends to 0, while

- the fraction of the points p for which the *closest* is one of the *edges* tends to 1.

Similar arguments can be repeated for any dimension. For the same noise level σ, when d increases, the distance $\varepsilon = \sigma \cdot \sqrt{d}$ also increases, and therefore, for large d, for "almost all" observed points $\vec{o}$, the reconstructed signal is one of the extreme points of the a priori set A.

Which value a should we choose for this modified intermediate dependence: specifics. Let us apply the above general idea to our specific case. In this case, the value a can take any values from 0 to ∞, so the extreme cases are $a = 0$ and $a = \infty$.

Of course, literally taking $a = 0$ or $a = \infty$ makes no sense, since for each value $x + x_0$, the power $(x + x_0)^0$ is simply equal to 1 – i.e., does not depend on x at all, while $(x+x_0)^\infty$ is either 0 (if $|x + x_0| < 1$) or infinity (if $|x + x_0| > 1$). So, to get non-trivial expressions, instead of directly substituting $a = 0$ or $a = \infty$ into the above formula, we need to consider limit cases when $a \to 0$ or $a \to \infty$.

Let us first consider the case $a \to 0$. In general, we have

$$(x + x_0)^a = (\exp(\ln(x + x_0)))^a = \exp(a \cdot \ln(x + x_0)).$$

For small $a \approx 0$, we can expand this expression in Taylor series and keep only linear terms in this expression:

$$(x + x_0)^a \approx 1 + a \cdot \ln(x + x_0).$$

Thus, for small a, the expression (3) tends to a linear transformation of a logarithm:

$$y = c_0 + c_1 \cdot \ln(x + x_0). \tag{4}$$

The case when $a \to \infty$ can be obtained from this case if we take into account that when y is related to x by a formula (3) with some a, then x is related to y by a similar formula, but with an exponent $1/a$. When $a \to 0$, then $1/a \to \infty$. So, the limit dependence corresponding to $a \to \infty$ is the inverse of the dependencies corresponding to $a \to 0$, i.e., a linear transformation of the exponential function:

$$y = c_0 + c_1 \cdot \exp(k \cdot x). \tag{5}$$

What is the resulting dependence. We started with the case when we have several sequential transformations, all of which are power laws. In this case, the resulting dependence of y on x is still a power law. To get beyond the power laws, we decided to consider the case when on the intermediate transformations has a more general form (3) – and we argued that the most probable cases are extreme cases $a \to 0$ or $a \to \infty$ that are described by formulas (4) and (5). What will then be the resulting dependence between x and y?

Let us start with considering the case when the intermediate transformation is described by a logarithm formula (4). In this case,

- first, we have several power-law transformations, which, as we have learned, are equivalent to a single power-law transformation; as a result, the original value x is transformed into a new value $x_1 = a_1 \cdot x^{b_1}$ for some a_1 and b_1;
- then, to the resulting value x_1, we apply the logarithm transformation (3), resulting in
$$x_2 = c_0 + c_1 \cdot \ln(x_1 + x_0) = c_0 + c_1 \cdot \ln(a_1 \cdot x^{b_1} + x_0);$$
- finally, we again have several power-law transformations, which are equivalent to a single power-law transformation $y = a_3 \cdot x_2^{b_3}$ for some values a_3 and b_3, resulting in
$$y = a_3 \cdot (c_0 + c_1 \cdot \ln(a_1 \cdot x^{b_1} + x_0))^{b_3}. \tag{6}$$

Let us simplify this formula. Let us simplify this formula, to make it closer to the desired formula (1). First, we can represent $a_1 \cdot x^{b_1} + x_0$ as $c_2 \cdot (a'_1 \cdot x^{b_1} + e)$, where we denoted $c_1 \stackrel{\text{def}}{=} \dfrac{x_0}{e}$ and $a'_1 \stackrel{\text{def}}{=} \dfrac{a_1}{c_2} = \dfrac{a_1 \cdot e}{x_0}$. Then,

$$\ln(a_1 \cdot x^{b_1} + x_0) = \ln(c_2 \cdot (a'_1 \cdot x^{b_1} + e)) = \ln(c_2) + \ln(e + a'_1 \cdot x^{b_1}),$$

and thus,

$$c_0 + c_1 \cdot \ln(a_1 \cdot x^{b_1} + x_0) = c_0 + c_1 \cdot \ln(c_2) + c_1 \cdot \ln(e + a'_1 \cdot x^{b_1}),$$

i.e.,

$$c_0 + c_1 \cdot \ln(a_1 \cdot x^{b_1} + x_0) = c'_0 + c_1 \cdot \ln(e + a'_1 \cdot x^{b_1}),$$

where we denoted $c'_0 \stackrel{\text{def}}{=} c_0 + \ln(c_2)$. This expression, in turn, can be described as

$$c_0 + c_1 \cdot \ln(a_1 \cdot x^{b_1} + x_0) = c'_0 + c_1 \cdot \ln(e + a'_1 \cdot x^{b_1}) =$$
$$c_1 \cdot (\ln(e + a'_1 \cdot x^{b_1}) + c''_0),$$

where $c''_0 \stackrel{\text{def}}{=} c'_0/c_1$. Thus,

$$(c_0 + c_1 \cdot \ln(a_1 \cdot x^{b_1} + x_0))^{b_3} = c_1^{b_3} \cdot (\ln(e + a'_1 \cdot x^{b_1}) + c''_0)^{b_3}.$$

Multiplying both sides by a_3, we conclude that the formula (6) can be described in the following form

$$y = a'_3 \cdot (\ln(e + a'_1 \cdot x^{b_1}) + c''_0)^{b_3}, \tag{7}$$

where $a'_3 \stackrel{\text{def}}{=} a_3 \cdot c_1^{b_3}$.

This is (almost) exactly what we want. The empirical formula (1) can be viewed as a particular case of the above formula (7), with $c''_0 = 0$, $a'_3 = \text{const}$, $a_1 = a^{-b}$, and $b_3 = -c$.

Vice versa, any expression (7) with $c''_0 = 0$ has the form (1). So, we (almost) have what we want: a theoretically justified formula: the only difference is that our formula has one more parameter c''_0. Who knows, maybe empirically, we can find some non-zero value of this parameter for which this formula will be even more accurate than the original empirical formula (1)?

Comments.

- When we describe limit cases of scale-invariance, we had a choice:
  - we could have a logarithmic dependence, or
  - we could have the inverse (exponential) dependence.

Which dependence we choose depends on which of two quantities we consider as input and which as output. If instead of the dependence $\theta(h)$, we will consider the inverse dependence $h(\theta)$, then we will get exponential function instead of the logarithmic one. Which of the two dependencies $\theta(h)$ or $h(\theta)$ is logarithmic and which is exponential cannot be determined purely theoretically, since we assume the same scale-invariance property for both quantities; this must be determined empirically. In this particular case, the dependence $\theta(h)$ is logarithmic.

- The additional factor (1a) can also be explained the same way: as one can see, it is exactly one of the two limit cases of power law dependency: namely, the logarithmic limit case (4).

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