Rosenzweig, Equality, and Assignment

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Abstract

In his seminal book "The Star of Redemption", the renowned philosopher Franz Rosenzweig illustrated his ideas by the intuitive difference between mathematical statements $A = B$ and $B = A$. Of course, from the purely mathematical viewpoint, these two statements are always equivalent, so to a person trained in mathematics – even in simple school mathematics – this illustration is puzzling. What we show is that from the viewpoint of common folks, there is indeed a subtle difference between how people understand these two equalities. To us, the understanding of this difference helped us better understand Rosenzweig's ideas. But we believe that this difference has application way beyond those interested in Rosenzweig's philosophy: namely, it makes sense to take this subtle difference into account when teaching mathematics in school.

1 Formulation of the Problem

Rosenzweig's use of equality. In his seminar book "The Star of Redemption" [1], renowned philosopher and theologist Franz Rosenzweig spends a lot of time explaining his concepts by comparing them to the difference between mathematical statements $A = B$ and $B = A$.

But what does this difference mean? Of course, from the purely mathematical viewpoint, this explanation makes no sense: in mathematics, the statement $A = B$ means exactly the same as $B = A$.

What we do in this paper. In this paper, we show that the difference mentioned by Rosenzweig makes sense if we consider not the precise mathematical meaning of this formula, but rather our intuitive understanding of equality in simple mathematics.

Why this may be interesting. Honestly, very few people are familiar with Rosenzweig's philosophy, so why is all this interesting? To us, this study was

interesting because it enabled us to realize that there is a subtle difference between people's perception of statements $A = B$ and $B = A$. Understanding this intuitive difference may help teachers of mathematics.

The teachers may have had similar different understandings when they were students themselves, but after studying math, they know that $A = B$ and $B = A$ means exactly the same thing – so they may not take into account the difference in student understanding when teaching.

2 Examples When There Is a Subtle Difference Between $A = B$ and $B = A$

First example. Suppose that we ask a student to solve an equation and to find the value of the unknown x. Suppose also that the correct answer is $x = 3$. In this case, if the student's answer is $x = 3$, the teacher is happy.

But what if the student's answer is $3 = x$? From the purely mathematical viewpoint, this is an equivalent form of the same correct answer, the student did not make any mistake. However, intuitively, this does not sound exactly right.

Why is this? We will discuss the difference after we present a few more similar examples.

Second example. Suppose now that a teaches asks the students to multiply 2 and 3. The expected answer is $2 \cdot 3 = 6$.

From the purely mathematical viewpoint, this desired statement is equivalent to $6 = 2 \cdot 3$, but intuitively, this answer – while mathematically correct – does not sound as good.

So maybe a short part – like 6 here – should be in the right-hand side? Not really, as our next example shows.

Third example. Suppose that a teacher asks the students to represent 6 as the product of prime numbers. In this case, the expected answer is $6 = 2 \cdot 3$.

Again, from the purely mathematical viewpoint, this desired statement is equivalent to $2 \cdot 3 = 6$, but, intuitively, this alternative answer does not sound as good.

General explanation. In all these examples – and in most problems from school mathematics – we have some quantity, and we want to represent in a different form:

- in the first example, we know that there is a variable x, and we want to find the numerical value of this variable;
- in the second example, we know that we are interested in the product $2 \cdot 3$, and we want to find the numerical value of this product;
- in the third example, we know the number 6, and we want to represent this number as a product.

In all three example, the intuitive way of presenting the asnwer is:

- to place what we knew in the beginning in the left-hand side of the equality, and
- to place the solution in the right-hand side of this equality.

In other words, when we write $A = B$, we mean that:

- we previously had some information about A, and
- later, it turned out that this quantity A is equal to the expression B.

This understanding can be summarized by the usual notation $x =?$ ($2 \cdot 3 =?$, $6 =?$) that is used to formulate the problem.

In view of this understanding, the formulas $A = B$ and $B = A$ indeed have different meanings, as was illustrated by the expressions $2 \cdot 3 = 6$ and $6 = 2 \cdot 3$:

- $A = B$ means that we knew A, and now we learned that it is equal to B; in the example $2 \cdot 3 = 6$, we knew that we need to multiply 2 by 3, and we learned that the result is 6;
- on the other, $B = A$ means that we knew B, and now we learned that it is equal to A; in the example, $6 = 2 \cdot 3$, we knew the number 6, and now we learned that it can be represented as the product of 2 and 3.

This understanding is somewhat related to assignment operation in programming languages. In many programming languages such as C, C++, Java, etc., the equal sign is not used to describe equality, it is used to describe assignment.

In this case, the statement $x = 3$ means that to the variable x, we assign a new value 3. From this viewpoint, $3 = x$ does not make any sense: the number 3 in the left-hand side is a constant, its value is fixed, we cannot assign any new value to this constant.

Of course, this analogy is not perfect: not everything that makes sense in a programming language makes sense in intuitive school math, and not every difference between $A = B$ and $B = A$ makes sense in a programming language.

- For example, in a programming language we can write $x = x + 1$, which means that we take the previous value of the variable x, add 1 to it, and place the new value into the same variable x. This is not how we understand equality in school mathematics; in school mathematics, the statement $x = x + 1$ would make no sense.
- On the other hand, the difference between $6 = 2 \cdot 3$ and $2 \cdot 3 = 6$ is not captured by the programming language analogy, since we cannot assign a new value to a constant 6.

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References

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