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SAME ARGUMENTS THAT PROVE EXISTENCE OF SINGULARITY PROVE EXISTENCE OF ACAUSALITY

Singularity. Most physicists believe that the physical space-time has a singularity. This belief is based on the fact that in General Relativity Theory – which has so far been supported by all the experiments and observations – almost all models have singularity; see, e.g., [1,2]. To be more precise, there are some models that do not have singularity – e.g., the flat Minkowski space-time with no matter in it – but such models are nowhere dense in the following sense:

- for each model with singularity, there exists a neighborhood (in a natural topology) in which all models have singularity, and
- In each neighborhood of each model without singularity, there is a model with singularity.

From this, the physicists conclude that models without singularity, while mathematically possible, are not physically possible – just like while it is mathematically possible that the Brownian molecules of all the molecules in a person's body will become vertical and the person will float in the air, no physicist would believe that this is physically possible.

Acausality. What we show in this talk is that similar arguments show that almost all models are acausal – i.e., have closed time-like curves – and thus, that it is reasonable to conclude that the actual physical space-time contains such a curve.

What is the natural topology: reminder. To explain our point, let us recall what is the natural topology on the set of all space-time models. In General Relativity Theory, each space-time model is char-

acterized by the metric, i.e., by the continuous field $g_{ij}(x)$ on a manifold M . A natural description of a neighborhood of this model is when we select an open subset S in M – whose closure is compact – and a positive real number $d > 0$. This neighborhood contains all the space-time models in which there is an area S' diffeomorphic to S , in which all the components of the metric field differ from the S 's values by no more than d .

How to prove the main result. Let us first show that all the models in some neighborhood of an acausal model are also acausal. Indeed, acausal means that there is a closed time-like curve, i.e., a curve $x(s)$, where s is proper time along this curve, for which the expression $g_{ij}(x)e_i(s)e_j(s)$ is positive for all s , where $e_i(s)$ denotes dx_i/ds . Since the metric field is continuous, this means that the minimum of the above expression over all s is positive. Thus, for a sufficiently small d , if another metric h_{ij} is d -close to g_{ij} , then the value $h_{ij}(x)e_i(s)e_j(s)$ will also be positive. In other words, the exact same curve will be closed in the new metric as well.

Suppose now that we have a causal model M , i.e., a model without any closed time-like curves, and let the pair (S,d) describe a neighborhood of this model. Let us show that this neighborhood contains an acausal model. Indeed, since the model is causal it cannot be compact – since in a compact space-time every time-like curve cannot go on infinitely, so it must loop on itself. Thus, there are points in M which are outside the compact closure C of the set S . We can thus artificially add a small time-like curve at one of these points – and thus, get an acausal space-time in the given neighborhood of the original model M .

Conclusion. The above result shows that we need to be honest, and select one of the following two options:

- The first option is to conclude that the actual space-time is acausal – just like we conclude that the actual space-time has a singularity.
- The second option is to still believe – as most physicists do – that the actual space-time is causal, but then explicitly understand that this needs to be presented as an additional postulate. We cannot simply

dismiss acausal space-time models as unusual cases, since, as we have shown, almost all space-time models are acausal, and it is causal models which are unusual cases.

References

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